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## DETERMINATION OF MEASUREMENT ERROR OF ELECTRICAL ENERGY QUALITY INDICATORS BY THE CORRELATION METHOD

*The problem of energy conservation is urgent not only for Ukraine but for the whole world. One of the main causes of non-productive energy costs in the energy sector is high losses in power grids. The problem of electricity quality is the focus of many researchers and practitioners of energy and electrification. The co-supply of single-phase and three-phase electrical energy receivers from a three-phase four-wire network is widespread in the power supply systems of industrial enterprises. In such cases, there is almost always a phase asymmetry of the load, as well as significant currents in the zero wire. These factors lead to a decrease in the quality of the voltage and are the cause of economic damage. Organization of hardware control of electricity quality indicators, organizational and economic mechanism of influence on the cause of deterioration of electricity quality, development of methods and technical methods of distortion are important for normalization of electricity quality. The error of the correlation method for measuring the unbalanced voltage of a three-phase network based on the preliminary conversion of a three-phase voltage to a two-phase voltage with subsequent decomposition into orthogonal components using a discrete Fourier transform is estimated.*

*Keywords: electrical energy, power quality, power system harmonics, calculation methods, asymmetry.*

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## ВИЗНАЧЕННЯ ПОХИБКИ ВИМІРЮВАННЯ ПОКАЗНИКІВ ЯКОСТІ ЕЛЕКТРИЧНОЇ ЕНЕРГІЇ КОРЕЛЯЦІЙНИМ МЕТОДОМ

*На сучасному етапі розвитку промисловості проблема енергозбереження становиться актуальною не лише для України, а й для всього світу. Одна з головних причин невиробничих затрат енергії в енергетиці це високі втрати в електричних мережах. Проблема якості електроенергії знаходиться у центрі уваги багатьох дослідників і практичних працівників в області енергетики й електрифікації. Спільне живлення однофазних і трифазних приймачів електричної енергії від трифазної чотирипровідної мережі знаходить широке поширення в системах електропостачання промислових підприємств. Трифазний контур, як правило, більш економічний, ніж еквівалентний двопровідний однофазний ланцюг з такою самою напругою лінії та заземлення, оскільки для передачі заданої кількості електричної енергії використовується менше матеріалу провідника. У таких випадках практично завжди виникає несиметрія навантаження за фазами, а також значні струми в нульовому проводі. Ці фактори призводять до зниження якості напруги й є причиною народногосподарського збитку. Для підвищення якості та нормалізації електроенергії важливими є організація апаратного контролю показників якості електроенергії, організаційно-економічний механізм впливу на причину погіршення якості електроенергії, розробка методів та технічних методів усунення спотворень електроенергії. Однак в реальному світі неможливо ідеально збалансувати міжфазну напругу в трифазному контурі. Напруга між лініями в трифазному контурі зазвичай змінюється на кілька вольт, але різниця більше одного відсотка може пошкодити двигуни та інше обладнання. Несбалансована напруга викликає незбалансований струм в обмотках двигуна; невідновлені струми викликають збільшення струму принаймні до однієї з обмоток та підвищення температури цієї обмотки. Такі підвищені температури скорочують термін експлуатації двигуна чи іншого обладнання, що призводить до передчасного виходу з ладу обладнання. Тому для контролю несиметрії напруги використовують цифрові методи вимірювання, що засновані на обробці миттєвих значень напруги. Для цих цілей в роботі оцінюється похибка кореляційного методу вимірювання несиметрії напруги трифазної мережі на основі попереднього перетворення трифазної напруги у двофазну напругу з подальшим розкладанням на ортогональні компоненти за допомогою дискретного перетворення Фур'є. Показана можливість визначення відносної похибки квантування напруги зворотної послідовності, що вноситься перешкодами в цифровий метод вимірювання несиметрії напруг.*

*Ключові слова: електрична енергія, якість електроенергії, гармоніки енергосистеми, методи розрахунку, несиметрія.*

### Introduction

One of the main conditions for ensuring the proper functioning of electrical equipment is the supply of electricity, the parameters of which meet certain requirements for its quality. The main indicators of electricity quality are related to such parameters as frequency and voltage deviation, voltage fluctuations, non-sinusoidal and voltage asymmetry. To avoid prolonged malfunction of electrical equipment, the main indicators of electricity quality should not go beyond their normal values.

Thus, the growth in the production and consumption of electricity makes ever higher demands on the accuracy of its metering. One of the ways to increase accuracy is related to taking into account errors in real working conditions. The purpose of this work is to evaluate the relative error of the quantization of the voltage of the negative sequence introduced by the noise into the digital method of measuring voltage unbalance.

### Theoretical basis

The development of society is accompanied by an increase in electricity consumption. Intensification of production, development of electrical engineering causes a sharp increase in energy intensity and concentration of loads. There is a qualitative and quantitative evolution of industrial consumers. The number of nonlinear, asymmetric, sharply varying industrial electricity consumers is increasing. Damage from the power supply breakdown, reasons for the output of the power quality parameters at normalized values and the resulting damage, these issues became not abstract. All this requires a comprehensive approach to energy quality issues.

The quality of electric energy is one of the most important requirements for the system of production, transmission and consumption of electricity. Deterioration in the quality of electric energy leads to a decrease in the reliability of power supply, an increase in energy losses, a deterioration in the quality and a decrease in the number of products.

Asymmetry of voltage is one of the main indicators of the quality of electric energy. Considering the main standards on the quality of electricity [1-3], we can derive a single asymmetry term as an indicator of the quality of electric energy: the state of a three-phase alternating current power supply system in which the rms values of the main components of the interfacial voltages or the phase shift angles between the main components of the interfacial voltages are not equal between each other.

The symmetric three-phase system of voltages is characterized the same at the modul and the phase of tensions in all three phases. At asymmetric modes of voltage in different phases are not equal.

Asymmetric modes in electrical networks arise for the following reasons:

1. not identical loads in different phases;
2. incomplete-phase operation of lines or other elements in the network;
3. different line parameters in different phases.

There are two types of asymmetry: systematic and random. Systematic asymmetry is caused by the non-uniform constant overload of one of the phases, the random asymmetry corresponds to the unconstant loads, at which different phases are overloaded at different times depending on random factors.

Asymmetry adversely affects the operating and technical and economic characteristics of rotating electric machines. The direct sequence current in the stator creates a magnetic field that rotates at a synchronous frequency in the direction of rotation of the rotor. The reverse sequence currents in the stator create a magnetic field rotating relative to the double synchronous frequency rotor in the direction opposite to rotation. Due to these double frequency currents, an electromagnetic torque and additional heating occur in the electric machine, resulting in a shorter insulation life.

Asynchronous motors have additional losses in the stator [13]. In some cases, it is necessary to increase the rated power of electric motors when designing, unless special measures are taken to symmetry voltage. In synchronous machines, in addition to the additional losses and heating of the stator and rotor, dangerous vibrations can start. Asymmetry shortens the life of transformer isolation and synchronous motors reduce the generation of reactive power [4].

The total loss caused by asymmetry in industrial networks includes the cost of additional losses of electricity, increase of deductions for renovation from capital costs, technological damage, damage caused by the decrease in the luminous flux of lamps installed in phases with low voltage, and the shortening of the service life of the lamps, installed in phases with high voltage, loss due to the reactive power generated by synchronous motors.

According to standard [1], the asymmetry coefficient of voltages in percent is determined from the expression

$$\varepsilon_2 = (U_2/U_n)100\%, \quad (1)$$

where  $U_2$  - reverse sequence voltage;  $U_n$  - nominal voltage value.

It follows from (1) that the control of asymmetry of voltage is reduced to measuring the voltage of the reverse sequence. To control the asymmetry of voltages, digital measurement methods based on the processing of instantaneous values of voltages are used [5-6]. Control of asymmetry of voltage is complicated by the fact that in real signals there is always interference, the value and statistical characteristics of which ultimately determine the error of asymmetry. Such interferences can be higher harmonics, instrumental errors and measurement errors of instantaneous voltage values [6].

### Results

When operating the power supply systems, the asymmetry of the voltages of the three-phase network is monitored. For these purposes, it is most promising to use digital control methods [7-9]. Digital methods are characterized by sampling and quantization errors of input signals.

Measuring the asymmetry of voltages of a three-phase network is complicated by the fact that in real power systems three-phase networks are "clogged" by higher harmonics [6]. At the non-sinusoidal form of the three-phase voltage, the asymmetry is found in the fundamental harmonic. Correlative signal processing methods based on discrete Fourier transforms are allowed to distinguish the fundamental harmonic with minimal error and in real time [9-10].

We estimate the error of the correlation method of measuring the asymmetry of voltages of a three-phase network based on the preliminary transformation of a three-phase network into two-phase, with the subsequent decomposition of each phase into orthogonal components by means of a discrete Fourier transform [9-11]. In this

method, the inverse sequence is described by an expression that allows us to calculate the quadrature components of the converted three-phase voltage into two-phase:

$$U_2^2 = \frac{1}{9}(V_x^2 + V_y^2 + W_x^2 + W_y^2 - 2V_yW_x + 2V_xW_y). \tag{2}$$

Converting (2), we write it in a more compact form

$$U_2^2 = \frac{1}{9}[(V_x + W_y)^2 + (W_x - V_y)^2], \tag{3}$$

where  $V_x, W_x, V_y, W_y$  - quadrature components of voltage,

$$\begin{aligned} V_y &= \frac{2}{m} \sum_{i=0}^{m-1} v(t_i) \cos \omega t_i, \\ V_x &= \frac{2}{m} \sum_{i=0}^{m-1} v(t_i) \sin \omega t_i, \\ W_y &= \frac{2}{m} \sum_{i=0}^{m-1} w(t_i) \cos \omega t_i, \\ W_x &= \frac{2}{m} \sum_{i=0}^{m-1} w(t_i) \sin \omega t_i, \end{aligned}$$

where  $v(t_i)$  and  $w(t_i)$  - instantaneous values of two-phase voltage converted from three-phase;  $t_i = 2\pi i / (m\omega)$  - moments of sampling voltages  $v$  and  $w$ ;  $m$  - number of sampling points.

Denoting  $(V_x + W_y) = \Phi_1$  and  $(W_x - V_y) = \Phi_2$ , we represent (3) in the form

$$U_2^2 = \frac{1}{9}[\Phi_1^2 + \Phi_2^2].$$

Since the negative sequence voltage is a function of  $U_2^2 = f(\Phi_1, \Phi_2)$ , we determine its absolute error

$$\Delta U_2^2 = \frac{\partial f}{\partial \Phi_1} \Delta \Phi_1 + \frac{\partial f}{\partial \Phi_2} \Delta \Phi_2, \tag{4}$$

where from  $\Delta U_2^2 = \frac{2}{9}[\Phi_1 \Delta \Phi_1 + \Phi_2 \Delta \Phi_2]$ .

On the other hand  $\Delta(U_2^2) = 2U_2 \Delta U_2$ . Thus

$$\Delta U_2 = \frac{1}{9U_2} [\Phi_1 \Delta \Phi_1 + \Phi_2 \Delta \Phi_2],$$

where  $\Delta \Phi_1 = \Delta V_x + \Delta W_y$  and  $\Delta \Phi_2 = \Delta W_x - \Delta V_y$ , and the quadrature components of the phase errors are determined from the expressions

$$\left\{ \begin{aligned} \Delta V_y &= \frac{2}{m} \sum_{i=0}^{m-1} \Delta v(t_i) \cos \omega t_i \\ \Delta V_x &= \frac{2}{m} \sum_{i=0}^{m-1} \Delta v(t_i) \sin \omega t_i \\ \Delta W_y &= \frac{2}{m} \sum_{i=0}^{m-1} \Delta w(t_i) \cos \omega t_i \\ \Delta W_x &= \frac{2}{m} \sum_{i=0}^{m-1} \Delta w(t_i) \sin \omega t_i \end{aligned} \right. , \tag{5}$$

$\Delta v(t_i)$  and  $\Delta w(t_i)$  - instantaneous values of phase quantization errors  $V$  and  $W$ .

Believing that the errors  $\Delta v(t_i)$  and  $\Delta w(t_i)$  is small, in (4), we can restrict ourselves to the first term in the Taylor series expansion.

For the mathematical expectation of the error  $\langle \Delta U_2 \rangle$  we write the expression

$$\langle \Delta U_2 \rangle = \frac{1}{9U_2} (\Phi_1 \langle \Delta \Phi_1 \rangle + \Phi_2 \langle \Delta \Phi_2 \rangle).$$

The brackets  $\langle \ \rangle$  mean averaging by ensemble. The condition stationary of the process allows us to write the variance of the absolute quantization error of the input signal, determined by averaging over the ensemble with a mathematical expectation which equal zero:

$$\langle (\Delta U_2)^2 \rangle = \frac{1}{81U_2^2} (\Phi_1^2 \langle \Delta \Phi_1^2 \rangle + \Phi_2^2 \langle \Delta \Phi_2^2 \rangle + 2\Phi_1 \Phi_2 \langle \Delta \Phi_1 \Delta \Phi_2 \rangle). \tag{6}$$

We express the terms of formula (6) in terms of the quadrature components of errors the phase:

$$\begin{aligned} \langle \Delta \Phi_1 \Delta \Phi_2 \rangle &= (\Delta V_x + \Delta W_y)(\Delta W_x - \Delta V_y) = \\ &= \langle \Delta V_x \Delta W_x \rangle + \langle \Delta W_y \Delta W_x \rangle - \langle \Delta V_x \Delta V_y \rangle - \langle \Delta W_y \Delta V_y \rangle. \end{aligned} \tag{7}$$

We express the terms of expression (7) in terms of the instantaneous values of errors the phase

$$\langle \Delta V_x \Delta W_x \rangle = \frac{2}{m} \sum_{i=0}^{m-1} \sum_{k=0}^{m-1} \langle \Delta v(t_i) \rangle \sin \omega t_i \langle \Delta w(t_k) \rangle \sin \omega t_k.$$

Assuming the value of  $v(t_i)$  and  $w(t_i)$  are uncorrelate, we write

$$\langle \Delta v(t_i) \Delta w(t_k) \rangle = \langle \Delta v(t_i) \rangle \langle \Delta w(t_k) \rangle = 0,$$

then  $\langle \Delta V_x \Delta W_x \rangle = 0$ . Similarly, we obtain  $\langle \Delta W_y \Delta V_y \rangle = 0$ .

Another component of the expression (7):

$$\langle \Delta W_y \Delta W_x \rangle = \frac{2}{m} \sum_{i=0}^{m-1} \sum_{k=0}^{m-1} \langle \Delta w(t_i) \rangle \cos \omega t_i \langle \Delta w(t_k) \rangle \sin \omega t_k,$$

$$\text{as } \langle \Delta w(t_i) \Delta w(t_k) \rangle = \begin{cases} 0, & i \neq k, \\ \sigma_w^2, & i = k. \end{cases}$$

For stationary process  $\sigma_w^2 = const$ , then

$$\langle \Delta W_y \Delta W_x \rangle = \frac{2}{m} \sigma^2 \sum_{i=0}^{m-1} \cos \omega t_i \sin \omega t_i = 0.$$

Similarly, we find that  $\langle \Delta V_x \Delta V_y \rangle = 0$ . Substituting the obtained expressions in (7) we have  $\langle \Delta \Phi_1 \Delta \Phi_2 \rangle = 0$ , i.e. there is no correlation between random variables  $\Delta \Phi_1$  and  $\Delta \Phi_2$ :

$$\langle \Delta \Phi_1^2 \rangle = \langle (\Delta V_x + \Delta W_y)^2 \rangle = \langle \Delta V_x^2 \rangle + \langle 2\Delta V_x \Delta W_y \rangle + \langle \Delta W_y^2 \rangle. \quad (8)$$

Substituting expressions (5) in (8), we find

$$\begin{cases} \langle \Delta V_x^2 \rangle = \frac{4}{m^2} \sum_{i=0}^{m-1} \langle \Delta v(t_i) \rangle \sin^2 \omega t_i \\ \langle \Delta W_y^2 \rangle = \frac{4}{m^2} \sum_{i=0}^{m-1} \langle \Delta w(t_i) \rangle \cos^2 \omega t_i \end{cases} \quad (9)$$

Assuming the process is stationary, we can write  $\sigma_v^2 \equiv \Delta v^2(t_i)$  and  $\sigma_w^2 \equiv \Delta w^2(t_i)$ . Then expressions (9) take the form

$$\langle \Delta V_x^2 \rangle = \frac{2}{m} \sigma_v^2; \quad \langle \Delta W_y^2 \rangle = \frac{2}{m} \sigma_w^2; \quad (10)$$

$$\langle 2\Delta V_x \Delta W_y \rangle = \frac{2}{m} \sum_{i=0}^{m-1} \sum_{k=0}^{m-1} \langle \Delta v(t_i) \rangle \sin \omega t_i \langle \Delta w(t_k) \rangle \cos \omega t_k.$$

Assuming  $v(t_i)$  and  $w(t_i)$  are uncorrelated quantities, we have

$$\langle \Delta w(t_i) \Delta w(t_k) \rangle = \langle \Delta v(t_i) \rangle \langle \Delta w(t_k) \rangle = 0,$$

i.e.  $\langle 2\Delta V_x \Delta W_y \rangle = 0$ .

Similarly, for the terms  $\langle \Delta \Phi_2^2 \rangle$  we find

$$\langle \Delta W_x^2 \rangle = \frac{2}{m} \sigma_w^2; \quad \langle \Delta V_y^2 \rangle = \frac{2}{m} \sigma_v^2. \quad (11)$$

Substituting expressions (10) and (11) in (6), we obtain

$$\langle (\Delta U_2)^2 \rangle = \frac{2}{81mU_2^2} [\Phi_1^2(\sigma_v^2 + \sigma_w^2) + \Phi_2^2(\sigma_w^2 + \sigma_v^2)]. \quad (12)$$

If  $\sigma_v = \sigma_w = \sigma$ , then (12) takes the form

$$\langle (\Delta U_2)^2 \rangle = \frac{4\sigma^2}{81mU_2^2} (\Phi_1^2 + \Phi_2^2)$$

We rewrite the last expression taking into account formula (6):

$$\langle (\Delta U_2)^2 \rangle = \frac{4\sigma^2}{9m} \quad (13)$$

The variance of the error in quantizing the voltage of the negative sequence is determined by the expression (13). The relative error of the quantization of the voltage of the negative sequence is determined by the expression

$$\delta U_2 = \frac{\sqrt{\langle (\Delta U_2)^2 \rangle}}{U_2} \quad (14)$$

We substitute (13) into (14). Assuming that the asymmetry of the voltage of the three-phase network obeys the normal distribution law, the error  $\delta U_2$  is determined by the rule of three sigma:

$$\delta U_2 = 3 \sqrt{\frac{4\sigma^2}{9m}} / U_2 = \frac{2\sigma}{U_2 \sqrt{m}}$$

Multiplying the numerator and denominator of this expression by  $U_n$ , we find

$$\delta U_2 = \frac{2}{\sqrt{m}} \frac{\sigma U_n}{U_n U_2}, \quad (15)$$

where  $\frac{U_2}{U_n} = \varepsilon_2$  - voltage unbalance factor.

It should be noted that with a uniform distribution of the remainder from dividing the quantized quantity by the quantization step  $q$ , the quantity величина  $\sigma = 0.5q\sqrt{3}$  [12]. Then expression (15) takes the form

$$\delta U_2 = \frac{\delta_q}{\sqrt{3m}U_2}, \quad (16)$$

where  $\delta_q = q/U_n$  - relative quantization error.

### Summary & Conclusions

Asymmetry is a serious problem of electricity quality, with a negative impact mainly on low-voltage distribution networks. A successful solution to the problem of asymmetry, of course, leads to lower operating costs and, most importantly, energy loss.

The error of the correlation method of measuring the asymmetry of voltages of a three-phase network based on the preliminary conversion of a three-phase voltage into a two-phase with subsequent decomposition into orthogonal components by means of a discrete Fourier transform is performed in the work.

According to standard [1], the voltage unbalance factor should not exceed 2%. Given the voltage unbalance factor  $\varepsilon_2$  and knowing the digit of the ADC, the formula (16) can be used to calculate the relative error of quantization of the voltage of the reverse sequence  $\delta U_2$  at different numbers of sampling points  $m$ .

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