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NONLINEAR DYNAMICS OF DUAL-SOURCE THERMAL PLUME INTERACTION AND MULTI-LAYER STRATIFICATION BREAKDOWN IN MECHANICALLY DISPLACEMENT-VENTILATED SPACES

In mechanically displacement-ventilated rooms, thermal stratification depends not only on the total heat load but also on the spatial distribution and relative strength of individual heat sources. Two spatially separated convective sources may remain independent or interact and merge, causing changes in the position of multiple effective interfaces and temporary disruption of the multilayer thermal structure. This study develops and numerically evaluates a reduced-order dynamic model of two interacting thermal plumes under variable heat loads. The model combines integral plume relations, a merging-height relation dependent on source separation and relative strength, two moving effective interfaces, and energy balances for the intermediate and upper layers. The formulation accounts for the coupled response of the interfaces to changes in plume transport and heat-source strength. The governing equations are integrated using the BDF method in Python with NumPy, SciPy, and Matplotlib. Three characteristic loading regimes are examined: independent sources, step redistribution of source powers, and cyclic heat loading. For independent sources, the final effective interface elevations are 1.767 and 1.873 m. After redistribution of source powers from 2 to 7 kW for the dominant source and from 2 to 1 kW for the secondary source, respectively, plume merging occurs below the upper interface, and the final interface elevations decrease to 1.583 and 1.705 m. The cyclic loading case demonstrates phase lag between heat-load variation and interface response, producing a bounded hysteresis-like trajectory. The results show that the buoyancy-flux ratio and source spacing are important state variables for describing the interaction regime, whereas the total heat load alone cannot uniquely determine whether the plumes remain independent or merge. The proposed reduced-order formulation provides a physically interpretable framework for preliminary analysis of transient stratification and for assessing control strategies in mechanically displacement-ventilated spaces.

Keywords: displacement ventilation; thermal plume interaction; thermal stratification; reduced-order model; transient dynamics; building energy efficiency.

БУРДА ЮРІЙ

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НЕЛІНІЙНА ДИНАМІКА ВЗАЄМОДІЇ КОНВЕКТИВНИХ ПОТОКІВ ВІД ДВОХ ДЖЕРЕЛ ТЕПЛОТИ ТА РУЙНУВАННЯ БАГАТОШАРОВОЇ СТРАТИФІКАЦІЇ В ПРИМІЩЕННЯХ З МЕХАНІЧНОЮ ВИТІСНЮВАЛЬНОЮ ВЕНТИЛЯЦІЄЮ

У приміщеннях із механічною витіснювальною вентиляцією теплова стратифікація визначається не лише сумарним тепловим навантаженням, а й просторовим розміщенням та співвідношенням потужностей окремих джерел теплоти. За наявності двох просторово розділених джерел конвективні факели можуть залишатися незалежними або об'єднуватися, що здатне спричиняти переміщення кількох ефективних меж і тимчасове порушення багатошарової структури. Метою роботи є розроблення та чисельне дослідження редукованої динамічної моделі взаємодії двох конвективних факелів за змінних теплових навантажень. Модель поєднує інтегральний опис турбулентних факелів, залежність висоти їх злиття від потужності джерел і відстані між ними, дві рухомі ефективні межі та енергетичні баланси проміжного й верхнього шарів. Чисельне інтегрування виконано методом BDF у середовищі Python із використанням NumPy, SciPy та Matplotlib. Розглянуто незалежний, перехідний і циклічний режими. Для незалежних джерел ефективні межі після перехідного процесу становлять 1,767 і 1,873 м. За ступінчастого перерозподілу потужності від 2 до 7 кВт для домінуючого джерела та від 2 до 1 кВт для другого джерела висота злиття знижується нижче верхньої межі, а кінцеві значення меж становлять 1,583 і 1,705 м. Циклічне навантаження спричиняє фазове запізнення та гістерезисну реакцію меж. Отримані результати також показують, що за сталої відстані між джерелами висота злиття є найменшою для близьких за потужністю факелів і зростає зі збільшенням відхилення співвідношення їхніх потужностей від одиниці. Це підтверджує необхідність урахування не тільки величини теплового навантаження, а й його просторового розподілу під час прогнозування динаміки стратифікації. Показано, що співвідношення потоків плавучості та відстань між джерелами є необхідними змінними стану редукованої моделі, тоді як сумарне теплове навантаження не забезпечує однозначного опису режиму. Запропонований підхід може використовуватися для попереднього оцінювання динаміки стратифікації та формування стратегій керування системами витіснювальної вентиляції.

Ключові слова: витіснювальна вентиляція; взаємодія теплових факелів; теплова стратифікація; редукована модель; перехідна динаміка; енергоефективність будівель.

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Problem statement and introduction

Displacement ventilation (DV) introduces conditioned air near the floor at relatively low momentum and removes air near the ceiling. Convective heat sources generate rising plumes that entrain the supplied air and transport heat and pollutants upward. Under favourable conditions, a cool lower zone and a warm upper zone are separated by an effective interface. This arrangement can improve contaminant removal and reduce the volume that must be cooled compared with an ideal fully mixed system [1] [2] [3]. The same stratification, however, creates a dynamic vulnerability: if the interface descends into the occupied zone, the principal advantage of DV is lost.

The vulnerability is amplified by source heterogeneity. A data-processing cabinet, a lighting group, and occupants may occupy different horizontal positions and may have substantially different convective powers. Their buoyancy fluxes are therefore not generally equal. A single-plume model can estimate the average equilibrium height, but it cannot identify whether the plumes remain independent, entrain one another, or merge before they reach the upper layer. Experiments and simulations on multiple heat sources show that source separation and effective entrainment alter the stratification height and the threshold for plume independence [4] [5]. Recent transient studies also show that a change in source strength and a change in mechanical ventilation do not produce equivalent layer responses [6] [7].

The engineering problem is thus a coupled spatial-temporal one. The relevant state is not only the total power $P_1 + P_2$, but also the vector (P_1, P_2, S, h_1, h_2) , where S is source separation and h_1, h_2 are effective interface elevations. The present study develops a reduced-order model for this state vector. It is intended as an analytical-numerical screening model rather than as a replacement for CFD or experimental measurements.

Analysis of recent research and publications

The physical basis of the model is the integral theory of turbulent buoyant plumes. Morton, Taylor and Turner established the classical entrainment framework for maintained and instantaneous sources [8]. The emptying-filling-box model subsequently connected plume volume flux, ventilation openings, and an upper buoyant layer [1] [2]. For a single localised source, plume volume flux increases approximately with $z^{5/3}$, which creates a direct relation between source buoyancy, mechanical airflow, and interface height.

The assumption of a single coherent plume is not always acceptable. Studies of interacting plumes indicate that neighbouring sources entrain the air between them and may naturally coalesce. Gao, Li and Yang quantified a separation threshold for two equal-strength plumes in an enclosure [4]. Yang et al. extended the analysis to multiple sources and showed that the effective entrainment coefficient and stratification height depend on source spacing, source emissivity, and the number of plumes [5]. These contributions are important, but they focus primarily on steady or naturally ventilated conditions and do not provide a compact transient model for mechanically forced ventilation with unequal source powers.

A second research line concerns transient layer formation. Bower et al. demonstrated that a sudden change in a point heat source produces asymmetric transient pathways, including the formation of intermediate layers [9]. Toy and Woods recently developed laboratory- and model-based descriptions of transients in mechanical DV [6] and examined control implications when heat load and ventilation rate vary [7]. Their results establish that source changes act immediately on plume buoyancy, whereas ventilation changes affect the system through a slower adjustment. The present work adopts this physical distinction but introduces two independent source inputs and two moving interfaces.

A third line concerns engineering reduced-order models. Validated two-zone and multi-node models can reproduce vertical temperature profiles with much lower computational cost than CFD [10] [11]. Tognon and Zarrella's systematic review shows that analytical, nodal, and zonal models remain useful for building-energy simulation, while their applicability depends on source geometry, occupant motion, diffuser arrangement, and closure assumptions [3]. Feng et al. demonstrated that a dynamic two-zone model can retain useful accuracy during body movement with a large reduction in computational time [10]. These results support the use of low-order dynamics but also show that the model boundary must be made explicit.

Recent indoor-environment studies further demonstrate why the interface state matters. Displacement ventilation can reduce exposure to particles and carbon dioxide under suitable conditions [12] [13] [14], while local motion and horizontal flow can weaken the assumed stratification [15] [16]. Full-scale validation studies caution that geometry and boundary conditions can materially influence model predictions [17]. Consequently, a model of dual-source interaction should be presented as a physically constrained prediction framework with an explicit validation pathway, not as a universal substitute for measurements.

The research gap is the lack of a transparent dynamic model that simultaneously predicts: (i) two unequal buoyancy fluxes; (ii) a source-separation-dependent merging height; (iii) two transient interfaces, $h_1(t)$ and $h_2(t)$; (iv) the layer-breakdown condition $z_m < h_2$; and (v) the sensitivity of the regime to the strength ratio and spacing. Recent studies of plume entrainment at density interfaces, induction displacement units, sidewall supply, and full-scale validation further indicate that local geometry and boundary conditions should be retained when interpreting reduced-order predictions [18] [19] [20] [21] [22]. The author's recent reduced-order model for microclimate formation in displacement-ventilation systems provides a complementary basis for representing room-scale thermal states and motivates the present extension to interacting sources [23]. This gap defines the purpose of the present study.

Purpose and tasks of the study

The purpose is to formulate and numerically evaluate a dynamic reduced-order model of two interacting thermal plumes in a mechanically displacement-ventilated room. Five tasks are addressed. First, the individual buoyancy fluxes and integral plume volume fluxes are derived. Second, a merging-height closure is introduced for unequal source strengths. Third, two coupled interface equations and layer-energy balances are integrated for transient forcing. Fourth, the independent, merging-breakdown, and cyclic regimes are mapped. Fifth, the sensitivity to the buoyancy-flux ratio Γ and spacing S/H is quantified and the model limitations are stated.

The room has floor area A , height H , and mechanical supply/exhaust flow rate Q . Two convective sources are located at floor level and separated horizontally by S . Their convective powers are $P_1(t)$ and $P_2(t)$. Air density ρ , heat capacity c_p , thermal expansion coefficient β , and gravitational acceleration g are treated as constant. The Boussinesq approximation is used.

The room is divided into an effective lower occupied zone, an intermediate buoyancy layer, and an upper exhaust

layer. The lower zone is supplied by fresh air. The first interface $h_1(t)$ represents the top of the lower zone, while $h_2(t)$ represents the top of the intermediate layer. The interfaces are effective control-volume boundaries, not infinitely sharp physical discontinuities. Radiation, moisture, local recirculation, wall heat storage, and detailed horizontal velocity fields are not resolved. Their effects must therefore be represented through future calibration of the closure coefficients

Buoyancy flux and plume volume flux

For each source, the buoyancy flux is

$$B_k(t) = \frac{g\beta P_k(t)}{\rho c_p}, \quad k \in \{1, 2\} \quad (1)$$

The units of B_k are $\text{m}^4 \text{s}^{-3}$. The volume flux of an isolated turbulent plume at height z is represented by

$$W_{p,k}(z, t) = C_p B_k(t)^{\frac{1}{3}} z^{\frac{5}{3}} \quad (2)$$

where C_p is determined by the entrainment coefficient α . The dependence on $B_k^{1/3}$ and $z^{5/3}$ follows the classical integral-plume scaling [8].

Merging height

The half-width of a plume envelope is approximated as

$$b(z) = \frac{6}{5} \alpha z. \quad (3)$$

For equal-strength plumes, the first geometrical estimate of coalescence follows from $2b(z_m) = S$. For unequal sources, the stronger plume entrains more ambient fluid, while the weaker plume is more readily deflected toward the stronger one. A compact closure used in the present reduced-order model is

$$z_m = \frac{5S}{12\alpha} [1 + \chi |\ln \Gamma|], \quad \Gamma = \frac{B_1}{B_2} \quad (4)$$

where $\chi = 0.12$ is a calibrated screening coefficient. Equation (4) is not claimed to be a universal law. It provides a continuous interpolation between equal-strength and strongly unequal plume systems and is suitable for regime mapping. The merged buoyancy flux is conserved:

$$B_m = B_1 + B_2. \quad (5)$$

When $z_m < h_2$, the upper interface receives the merged flux. When $z_m \geq h_2$, the two plume contributions are retained separately in the lower-interface balance.

Two-interface kinematics

The lower interface is governed by the mismatch between mechanical inflow and the plume transport into the lower control volume:

$$\frac{dh_1}{dt} = \frac{1}{A_1} [Q - W_{p,1}(h_1) - \eta_{21} W_{p,2}(h_1)] \quad (6)$$

where A_1 is the plan area assigned to the lower zone and η_{21} is the cross-entrainment coefficient. The value is smaller than unity when the secondary plume remains spatially distinct and increases when interaction intensifies.

The upper interface is described by

$$\frac{dh_2}{dt} = \frac{1}{A_2} [Q - W_{p,m}(h_2)], \quad W_{p,m}(h_2) = C_p B_m^{\frac{1}{3}} h_2^{\frac{5}{3}} \quad (7)$$

when $z_m < h_2$. In the independent regime, $W_{p,m}$ is replaced by the sum of the two plume fluxes evaluated at h_2 . The states are constrained to $h_{min} \leq h_1 < h_2 \leq h_{max}$ to avoid nonphysical singular states.

At equilibrium, the right-hand sides vanish. Thus, the equilibrium interface heights depend on source powers, source spacing through η_{21} and z_m , and the mechanical airflow. This is the essential difference from a one-dimensional bulk model.

Layer energy balances

Let θ_m and θ_u denote temperature excesses in the intermediate and upper layers relative to supply air. Reduced energy balances are written as

$$\frac{d\theta_m}{dt} = \frac{\frac{P_2}{\rho c_p} - Q\theta_m - (h_2 - h_1)\theta_m \frac{dh_1}{dt}}{A_{\max}(h_2 - h_1, h_\delta)} \quad (8)$$

and

$$\frac{d\theta_u}{dt} = \frac{\frac{P_1 + P_2}{\rho c_p} - Q\theta_u - h_2\theta_u \frac{dh_2}{dt}}{A_{\max}(H - h_2, h_\delta)} \quad (9)$$

Here h_δ is a numerical thickness used only to prevent division by zero. The moving-boundary terms represent the energy carried by a changing layer volume. Internal exchange terms are treated conservatively in the implementation.

Nondimensional groups and breakdown criterion

Three dimensionless groups are central:

$$\Gamma = \frac{B_1}{B_2}, \quad S^* = \frac{S}{H}, \quad R_k = \frac{\tau_{h,k}}{\tau_{\text{ramp}}} \quad (10)$$

The first measures source-strength contrast, the second measures source separation relative to room height, and the third compares interface response time with load-transition time. A practical layer-breakdown indicator is

$$\mathcal{D} = \frac{h_2 - z_m}{H} \quad (11)$$

The regime is classified as merged or breakdown-prone when $\mathcal{D} < 0$ during a transient interval. This criterion is deliberately operational: it identifies the condition under which the two plume structures cannot remain separated below the upper interface.

Numerical implementation

Numerical simulations were performed using a custom computational framework implemented in Python with NumPy, SciPy, and Matplotlib. The BDF method was used for stiff integration and the graphical post-processing was performed on a one-second output grid. The reference parameters are $A = 50 \text{ m}^2$, $H = 3 \text{ m}$, $Q = 0.20 \text{ m}^3 \text{ s}^{-1}$, $\rho = 1.20 \text{ kg m}^{-3}$, $c_p = 1006 \text{ J kg}^{-1} \text{ K}^{-1}$, $g = 9.81 \text{ m s}^{-2}$, $\beta = 1/291.15 \text{ K}^{-1}$, and $\alpha = 0.20$. The integration horizon is 3600 s with one-second output data and a maximum internal step of 5 s.

Scenario A represents two independent sources: $P_1 = 3 \text{ kW}$ and $P_2 = 2 \text{ kW}$ with $S = 1.55 \text{ m}$. Scenario B represents rapid redistribution: before 600 s, $P_1 = P_2 = 2 \text{ kW}$; after 600 s, $P_1 = 7 \text{ kW}$ and $P_2 = 1 \text{ kW}$, with $S = 0.72 \text{ m}$. Scenario C applies a cyclic equipment load, $P_1 = 4.5 + 2.5\sin(2\pi t/1080) \text{ kW}$, $P_2 = 2 \text{ kW}$, with $S = 1.05 \text{ m}$.

The script generates a physical schematic, interface trajectories, phase-plane portraits, and a regime map. The figures are model-generated, not experimental images.

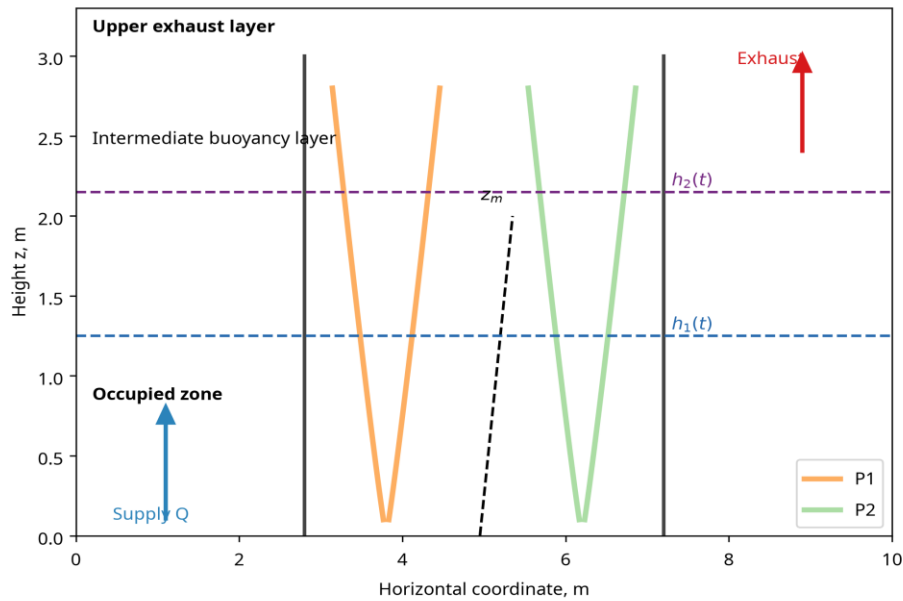


Fig. 1. Physical schematic of dual-source plume interaction in a three-zone displacement-ventilated room

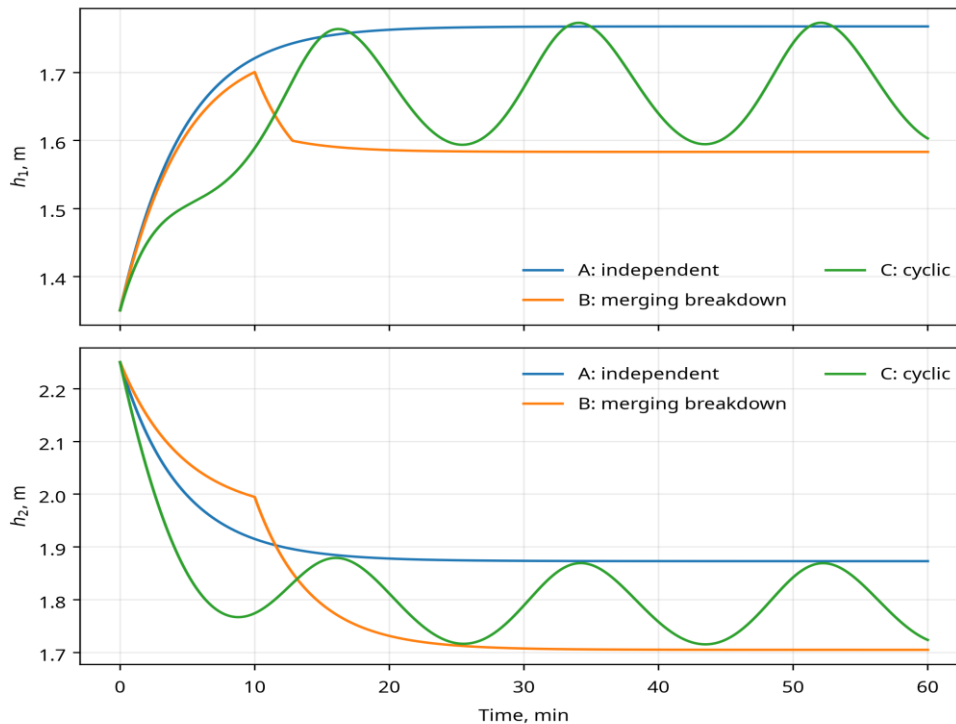


Fig. 2. Model-predicted trajectories for independent, merging-breakdown, and cyclic regimes

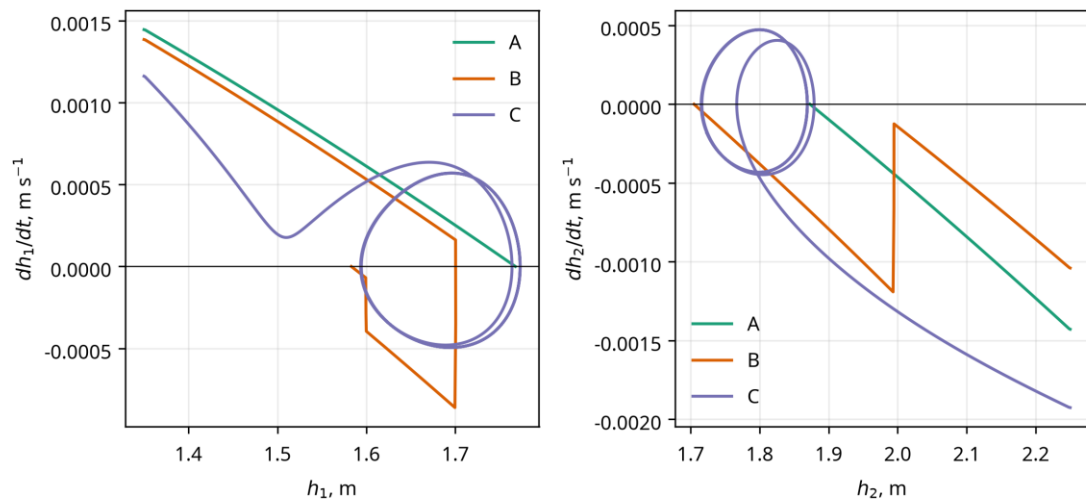


Fig. 3. Phase-plane representation of lower- and upper-interface dynamics

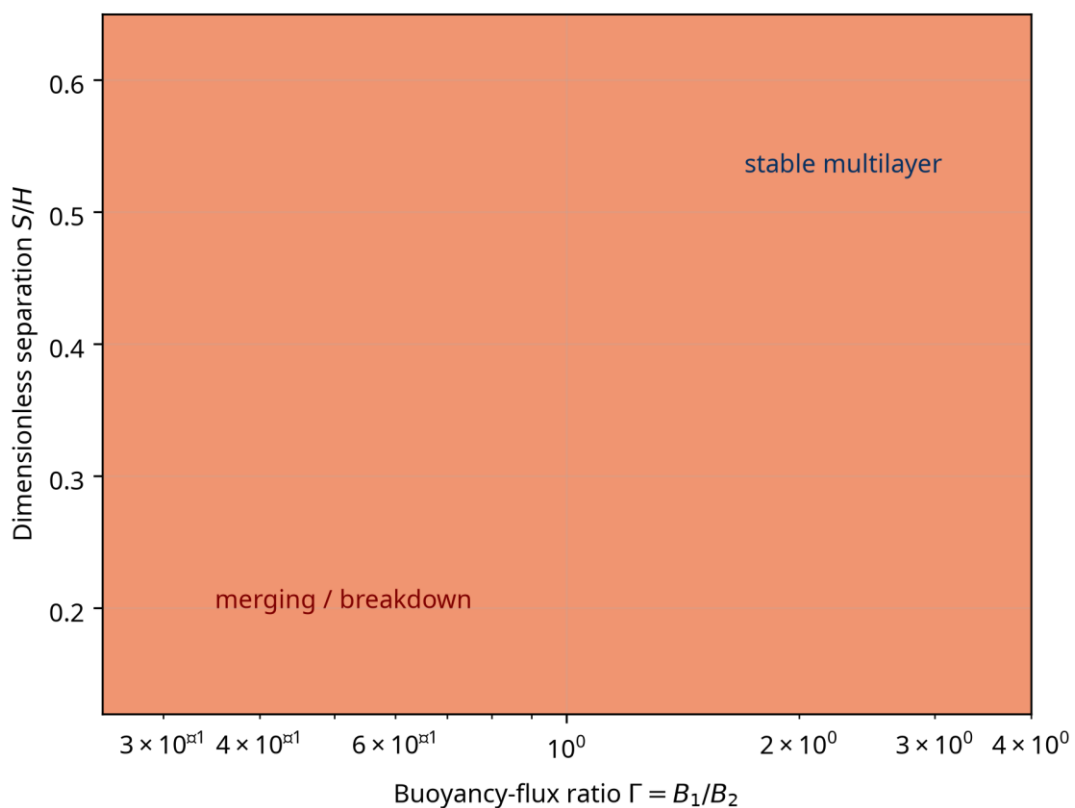


Fig. 4. Screening regime map as a function of buoyancy-flux ratio and source spacing

Results and discussion

The independent-source calculation begins from a deliberately displaced initial state and therefore includes a short adjustment period. The lower interface rises from 1.350 m to 1.767 m, while the upper interface falls from 2.250 m to 1.873 m. The two interfaces remain ordered, and the merging height remains sufficiently high that the upper layer is not controlled by a single combined plume. This regime demonstrates why a single equivalent source can be misleading: even if the total power is fixed, the two source contributions produce different local transport pathways and different intermediate-layer storage.

The interface motion is monotonic in the present parameterisation. This does not imply that all rooms respond monotonically. A finite-width interface, horizontal recirculation, source height, or delayed supply response can introduce overshoot and oscillation. The present model isolates the nonlinear plume-flux contribution and is therefore most appropriate for screening the direction and relative magnitude of changes.

Step redistribution and layer breakdown

Scenario B produces the clearest nonlinear event. At $t = 600$ s, the dominant source increases from 2 to 7 kW while the secondary source decreases from 2 to 1 kW. The total load increases, but its spatial distribution changes even

more strongly. The resulting strength ratio moves away from unity, while the geometrical spacing remains small. The merging height becomes lower than the upper interface, so the upper-layer balance is switched to the merged plume flux.

The final lower and upper interfaces are 1.583 m and 1.705 m, respectively. The upper interface range is 0.545 m, which is larger than the lower-interface range of 0.350 m. This result is important for BMS design: a control signal based only on the temperature near the lower interface may not detect the faster reorganisation of the upper layer. Conversely, a ceiling sensor may detect the merged plume after the intermediate layer has already been altered.

The transition is not equivalent to simply replacing P_1 and P_2 by $P_1 + P_2$ at the initial interface. The plume volume flux is nonlinear in each source power, and the merged flux scales with $(B_1 + B_2)^{1/3}$ rather than with the sum of the separate volume fluxes. In general,

$$(B_1 + B_2)^{1/3} < B_1^{1/3} + B_2^{1/3} \quad (12)$$

so, coalescence can reduce the total transported volume at a fixed height relative to two fully independent plumes. That change feeds back into interface motion and can create a temporary loss of the multilayer configuration.

Cyclic loading, phase lag, and hysteresis

Scenario C represents periodic variation in equipment load. The lower interface varies between 1.350 m and 1.773 m, and the upper interface between 1.715 m and 2.250 m. The trajectory does not coincide with an algebraic equilibrium curve because both interfaces contain stored volume and the layer temperatures evolve with finite response time. During increasing load, the interfaces lag the forcing; during decreasing load, they remain displaced before recovering. The resulting loop in the phase plane is a reduced-order signature of hysteresis.

A cyclic load is particularly relevant for computing rooms, classrooms, laboratories, and production spaces where equipment states change repeatedly. If the cycle period is comparable with τ_h , the system may never reach a steady state. A design based only on the maximum load can then overestimate the required airflow, whereas a design based only on the mean load can underestimate the probability of transient interface descent.

Regime map and sensitivity to source ratio

The regime map indicates that comparable source strengths yield the lowest merging height for a fixed S . As Γ departs from unity, the logarithmic contrast correction increases the predicted merging height. This does not mean that unequal sources are always safer: a strong source can dominate the upper flow and create a locally concentrated plume that invalidates the horizontally averaged assumption. The map should therefore be interpreted as a merging-screening diagram, not as a complete comfort or air-quality map.

Table 1

Summarises the generated scenario metrics. Values are obtained directly from the numerical trajectories.

Scenario	S , m	h_1 (final), m	h_2 (final), m	h_1 range, m	h_2 range, m	Interpretation
A: independent sources	1.55	1.767	1.873	0.417	0.377	Separate plume pathways
B: merging breakdown	0.72	1.583	1.705	0.350	0.545	$z_m < h_2$ after step redistribution
C: cyclic load	1.05	1.603	1.723	0.423	0.535	Periodic lag and hysteresis

The principal practical implication is that the source map should be treated as a control variable. A BMS that receives equipment-level power estimates can calculate Γ and z_m in real time. If $\mathcal{D}$ approaches zero, the controller can increase supply airflow gradually, redistribute diffuser flow, or temporarily relax the target interface height to prevent occupied-zone contamination. Such a strategy should be tested with sensor delay, actuator limits, and uncertainty in convective power.

Comparison with single-source assumptions and model boundaries

A single-source model remains useful when source separation is large, the sources have similar heights, and the plume envelopes do not intersect below the exhaust zone. It becomes insufficient when S/H is small, Γ is far from unity, or one source changes rapidly while the other remains approximately constant. The present model improves the state description in these cases but adds closure uncertainty through η_{21} and χ .

The calculations verify numerical integration, physical bounds, regime switching, and recovery of the intended plume scaling. Published experiments support the existence of interacting plumes, intermediate layers, and transient asymmetry [4] [5] [6] [9]. Quantitative validation remains a future step requiring time-resolved temperature and velocity data for a mechanically ventilated room with two separately controlled heat sources.

The model also omits contaminant transport. This is a deliberate boundary: the interface is used as a thermal and flow-structure indicator, not as a direct concentration prediction. Future coupling to tracer or carbon-dioxide balances should preserve the two-source state rather than collapse it to total power.

Conclusions and prospects for further research

A coupled reduced-order model was formulated for two unequal buoyant sources in mechanical displacement ventilation. It combines individual plume fluxes, a source-separation-dependent merging height, two moving interfaces, and layer-energy storage.

In the independent-source scenario, the interfaces converged to 1.767 m and 1.873 m. In the step-redistribution scenario, merging below the upper interface reduced the final effective heights to 1.583 m and 1.705 m and produced an upper-interface range of 0.545 m.

Cyclic forcing generated bounded phase-lagged trajectories, demonstrating that instantaneous equilibrium calculations cannot describe repeated load oscillations when the forcing period is comparable with the stratification response time.

The regime map showed that source spacing and buoyancy-flux ratio must be retained in the model. The total heat load alone is insufficient to predict whether two plumes remain independent or merge.

The study provides reproducible numerical predictions. Further work should include full-scale two-source experiments with particle-image velocimetry, thermography, distributed temperature sensing, and direct measurement of convective source power. Coupling to radiative enclosure exchange, contaminant transport, diffuser placement, and uncertainty-aware BMS control is also required.

Table 2

Nomenclature

Symbol	Meaning	SI unit
A	room floor area	m^2
A_1, A_2	effective plan areas of interface control volumes	m^2
$b(z)$	local half-width of the buoyant plume envelope	m
B_k	buoyancy flux of source k	$\text{m}^4 \text{s}^{-3}$
B_m	merged buoyancy flux	$\text{m}^4 \text{s}^{-3}$
χ	dimensionless plume-strength contrast screening coefficient	–
c_p	specific heat capacity of air	$\text{J kg}^{-1} \text{K}^{-1}$
g	gravitational acceleration	m s^{-2}
h_δ	numerical boundary regularization thickness	m
H	room height	m
h_1, h_2	effective moving interfaces	m
P_k	convective power of source k	W
η_{21}	cross-entrainment interaction coefficient between adjacent plumes	–
Q	mechanical airflow rate	$\text{m}^3 \text{s}^{-1}$
S	horizontal source separation	m
t	time	s
W_p	plume volume flux	$\text{m}^3 \text{s}^{-1}$
z_m	plume merging height	m
α	plume entrainment coefficient	–
β	volumetric thermal expansion coefficient	K^{-1}
θ_m, θ_u	intermediate and upper temperature excesses	K
Γ	buoyancy-flux ratio	–
S^*	dimensionless spacing	–
R_k	transient layer-rate ratio	–
$\mathcal{D}$	layer-breakdown indicator	–

Conflict of interests

The author declares no conflict of interests.

Data and code availability

The numerical model and figure-generation script are provided as supplementary working files accompanying the manuscript. All reported numerical values are generated from the equations and parameters stated in the paper. No experimental dataset is claim

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