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STRESSED STATE OF A TRANSVERSALLY ISOTROPIC MEDIUM WITH NON-CANONICAL CAVITIES

A spatial boundary value problem is considered for a transversely isotropic medium bounded by closed non-canonical surfaces obtained by rotating regular hexagons with rounded corners around one of their axes, with comprehensive stretching and compression. One of the effective approximate methods for studying the stress state of a deformable body with different boundary surfaces is a variant of the approximate method of boundary shape perturbation, developed and tested in the works of O. Guz and Yu. Nemish. In this case, boundary problems for an infinite medium bounded in the middle by non-canonical surfaces of revolution are formally reduced to a sequence of boundary problems for a medium with spherical surfaces. Articles [2, 3] are devoted to the use of the approximate method of boundary shape perturbation in solving boundary problems of mathematical elasticity theory. Numerical results are obtained for some transversely isotropic materials. The influence of material anisotropy on the stress concentration factor is analyzed. The boundary shape perturbation method was used to obtain solutions to the problem of the stress-strain state of thick layered shells of revolution [10]. In this case, the general solution of the equilibrium equations for an isotropic medium in a spherical coordinate system was used. The calculations allowed us to analyze the stress-strain state of shells under the action of internal and external pressure. The numerical results of the analysis of the stress-strain state of the shells can be added to the works of M. Leonov and K. Rusynka, V. Panasyuk, L. Berezhnytsky, S. Yarema, L. Ratych, and M. Stashchuk. [13-15], who evaluate the stress-strain state of composite materials with various defects. The latter use various criteria for the limit assessment of composites.

***Keywords**: transversely isotropic medium; non-canonical surfaces; boundary shape perturbation method; Legendre polynomials; thick layered non-canonical shells, close to spherical.*

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НАПРУЖЕНИЙ СТАН ТРАНСВЕРСАЛЬНО ІЗОТРОПНОГО СЕРЕДОВИЩА З НЕКАНОНІЧНИМИ ПОРОЖНИНАМИ

Розглядається просторова крайова задача для трансверсально ізотропного середовища, обмеженого замкнутими неканонічними поверхнями, отриманими внаслідок обертання правильних шестикутників з заокругленими кутами навколо однієї із своїх осей, при всесторонньому розтязі-стиску . Одним із ефективних наближених способів дослідження напруженого стану деформівного тіла з різними граничними поверхнями є варіант наближеного методу збурення форми границі, розробленого і апробованого в працях О.Гузя та Ю.Неміша. При цьому граничні задачі для нескінченого середовища, обмеженого в середині неканонічними поверхнями обертання, формально зводяться до послідовності крайових задач для середовища з сферичними поверхнями. Статті [2, 3] присвячені використанню наближеного методу збурення форми границі при розв'язуванні крайових задач математичної теорії пружності. Отримано числові результати для деяких трансверсально ізотропних матеріалів. Методом збурення форми границі отримано розв'язки задачі про напружено-деформований стан товстих шаруватих оболонок обертання [10]. При цьому використано загальний розв'язок рівнянь рівноваги для ізотропного середовища у сферичній системі координат. Розрахунки дозволили провести аналіз напружено-деформованого стану оболонок під дією внутрішнього і зовнішнього тиску. Отримані числові результати аналізу напружено-деформованого стану оболонок можна долучити до робіт М.Леонова і К.Русинка, В.Панасюка, Л.Бережницького, С.Яреми, Л.Ратича, М.Стащука. [13-15], які оцінюють напружено-деформований стан композиційних матеріалів з різними дефектами. В останніх використовуються різні критерії граничної оцінки композитів.

***Ключові слова:** трансверсально ізотропне середовище; неканонічні поверхні; метод збурення форми границі; поліноми Лежандра; товсті шаруваті неканонічні оболонки, близькі до сферичних.*

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Problem Statement

Among the main difficult classes of problems of the mathematical theory of elasticity are spatial boundary value problems. Mathematical difficulties in their solution arise in cases of both boundary surfaces and the influence of elastic properties of the deformable medium. Most of the exact solutions of spatial boundary value problems of the mathematical theory of elasticity for canonical domains are obtained by the method of separation of variables. It is based on general solutions of the basic equations in curvilinear orthogonal coordinate systems.

Analysis of known research results

Articles [2, 3] are devoted to the use of the approximate method of perturbing the boundary shape when solving boundary value problems of the mathematical theory of elasticity for figures close to the ellipsoid of revolution. The concentration of stresses in an isotropic medium bounded by non-canonical (close to spherical) cavities [1, 4, 5] and inclusions [6, 8] was investigated under all-round uniform deformation. Approximate solutions of external spatial problems were obtained for a transversely isotropic medium bounded by closed surfaces (conical, biconical and cylindrical) [9, 10] and in the form of a regular pentagon of revolution [11]. Thus, the concentration of stresses both on the surfaces and in their vicinity was investigated. The issue of the influence of the radius of curvature of the surface on the stress state of a homogeneous medium was analyzed in [12], where a rigid hypotrochoidal inclusion in an isotropic medium and a stress-free hypotrochoidal cavity in a transversely isotropic medium were considered.

Formulation of the article's objectives

The aim of the work is to investigate the stress state of a transversely isotropic medium bounded by closed surfaces of rotation under all-round deformation as well as to make a comparative analysis of the numerical values of normal stresses for materials whose elastic constants differ from the corresponding values in the isotropic case.

Presentation of the main material

An elastic homogeneous transversely isotropic medium (curvilinear anisotropy) with cavities in the form of regular hexagons of rotation with rounded corners is considered (Fig. 1, 2).

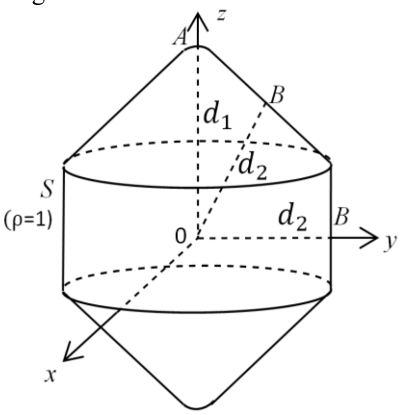

Fig. 1. **Non-canonical surface ($\mathcal{E} = 1/15$)**

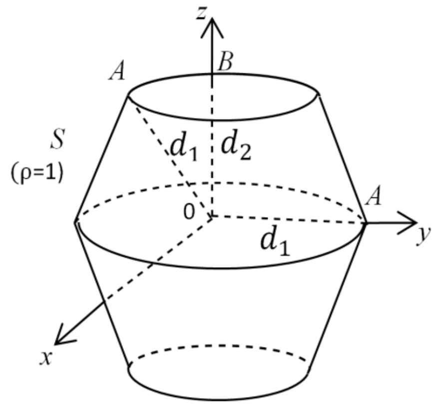

Fig. 2. Non-canonical surface ($\mathcal{E} = -1/15$)

Their surfaces *S* are formed by rotating the contour *G* around the axis *Oz*, the parametric equations of which have the form

$$z = r_0^{-1}\, Re(\omega(\xi))|_{\rho=1} = \cos\gamma + \varepsilon\cos 5\gamma,$$
$$R = r_0^{-1}\, \Im m(\omega(\xi))|_{\rho=1} = \sin\gamma - \varepsilon\sin 5\gamma\,. \tag{1}$$

They are written based on the function

$$z + iR = r_0^{-1}\,\omega(\xi) = \xi + \varepsilon\,\xi\, - 5 \quad (\xi = \rho i^{\gamma}, \quad \varepsilon = \pm 1/15), \tag{2}$$

which conformably maps the exterior of the unit circle of the plane ξ onto the exterior of the contour *G*. Here, $d_1 = 1 + |\varepsilon|$, $d_2 = 1 - |\varepsilon|$ are the distances from the center *O* to points *A* and *B*, respectively.

The study of the stress-strain state of the medium will be carried out by the approximate method of perturbing the shape of the boundary [1]. Therefore, the components $\sigma_{\rho\rho}$, $\sigma_{\gamma\gamma}$, $\sigma_{\varphi\varphi}$, $\sigma_{\rho\varphi}$, $\sigma_{\gamma\varphi}$ will be represented in the form of power series of a small parameter ε, i.e.

$$\sigma_{\rho\rho} = \sum_{j=0}^{\infty} \varepsilon^j\, \sigma_{\rho\rho}^{(j)}, \quad \ldots, \quad \sigma_{\gamma\varphi} = \sum_{j=0}^{\infty} \varepsilon^j\, \sigma_{\gamma\varphi}^{(j)} \tag{3}$$

Here the components $\sigma_{\rho\rho}^{(j)}, \ldots, \sigma_{\gamma\varphi}^{(j)}$ are found from the recurrence relations

$$\begin{pmatrix}\sigma_{\rho\rho}^{(j)}\\ \sigma_{\gamma\gamma}^{(i)}\end{pmatrix} = \sum_{m=0}^{j}\left[\Lambda_1^{(j-m)}\begin{pmatrix}\sigma_{rr}^{(m)}\\ \sigma_{\theta\theta}^{(m)}\end{pmatrix} \pm \Lambda_2^{(j-m)}\left(\sigma_{\theta\theta}^{(m)} - \sigma_{rr}^{(m)}\right) \pm \Lambda_3^{(j-m)}\sigma_{r\theta}^{(m)}\right];$$
$$\sigma_{\rho\gamma}^{(j)} = \sum_{m=0}^{j}\left[\Lambda_4^{(j-m)}\sigma_{r\theta}^{(m)} + \frac{1}{2}\Lambda_3^{(j-m)}\left(\sigma_{\theta\theta}^{(m)} - \sigma_{rr}^{(m)}\right)\right]; \qquad \sigma_{\varphi\varphi}^{(j)} = \sum_{m=0}^{j}\Lambda_i^{(j-m)}\sigma_{\alpha\alpha}^{(m)}; \tag{4}$$
$$\begin{pmatrix}\sigma_{\rho\varphi}^{(j)}\\ \sigma_{\gamma\varphi}^{(j)}\end{pmatrix} = \sum_{m=0}^{j}\left[\Lambda_5^{(j-m)}\begin{pmatrix}\sigma_{r\alpha}^{(m)}\\ \sigma_{\theta\alpha}^{(m)}\end{pmatrix} \pm \Lambda_6^{(j-m)}\begin{pmatrix}\sigma_{\theta\alpha}^{(m)}\\ \sigma_{ra}^{(m)}\end{pmatrix}\right].$$

The differential operators $\Lambda_i^{(j)}$ for a given boundary surface in the first three approximations have the form

$$\Lambda_1^{(0)} = \Lambda_4^{(0)} = 1; \qquad \Lambda_2^{(0)} = \Lambda_3^{(0)} = \Lambda_2^{(1)} = 0;$$
$$\Lambda_1^{(1)} = \Lambda_4^{(1)} = \frac{cos6\gamma}{\rho^5}\frac{\partial}{\partial\rho} - \frac{\sin 6\gamma}{\rho^6}\frac{\partial}{\partial\gamma};$$
$$\Lambda_3^{(1)} = \frac{12\sin 6\gamma}{\rho^6}; \qquad \Lambda_4^{(2)} = \Lambda_1^{(2)} - 2\Lambda_2^{(2)};$$
$$\Lambda_1^{(2)} = \frac{1+\cos 12\gamma}{4\rho^{10}}\frac{\partial^2}{\partial\rho^2} - \frac{\sin 12\gamma}{2\rho^{10}}\frac{\partial^2}{\partial\rho\partial\gamma}\frac{1}{\rho} + \frac{1-\cos 12\gamma}{4\rho^{12}}\left(\frac{\partial^2}{\partial\gamma^2} + \rho\frac{\partial}{\partial\rho}\right); \tag{5}$$
$$\Lambda_2^{(2)} = \frac{36(1-\cos 12\gamma)}{2\rho^{12}}; \qquad \Lambda_3^{(2)} = \frac{24\sin 12\gamma}{\rho^{12}} + \frac{6\sin 12\gamma}{\rho^{11}}\frac{\partial}{\partial\rho} - \frac{6(1-\cos 12\gamma)}{\rho^{12}}\frac{\partial}{\partial\gamma}.$$

A transversely isotropic medium under the action of uniform all-round forces p is considered.

$$\hat{\sigma}_{xx}^{(\infty)} = \hat{\sigma}_{yy}^{(\infty)} = \hat{\sigma}_{zz}^{(\infty)} = p \qquad \left(\hat{\sigma}_{xy}^{(\infty)} = \hat{\sigma}_{xz}^{(\infty)} = \hat{\sigma}_{yz}^{(\infty)} = 0\right). \tag{6}$$

In curvilinear orthogonal coordinates ρ, γ, φ for surfaces (ρ=1) the main stress state, i.e. for a medium without cavities, will be

$$\hat{\sigma}_{\rho\rho} = \hat{\sigma}_{\gamma\gamma} = \hat{\sigma}_{\varphi\varphi} = p \qquad \left(\hat{\sigma}_{\rho\gamma} = \hat{\sigma}_{\rho\varphi} = \hat{\sigma}_{\gamma\varphi} = 0\right). \tag{7}$$

Based on the assumption that the given cavities are free from stresses, the stress-strain state of the medium should be studied. With this formulation of the problem, the boundary conditions on the surface S (ρ=1) according to (4), (7) have the form

$$(\hat{\sigma}_{\rho\rho}^{(j)} + \sigma_{\rho\rho}^{(j)})_{\rho=1} = 0, \qquad (\hat{\sigma}_{\rho\gamma}^{(j)} + \sigma_{\rho\gamma}^{(j)})_{\rho=1} = 0 \quad (j \geq 0), \tag{8}$$

or equivalently

$$\sigma_{\rho\rho}^{(0)}|_{\rho=1} = -p; \qquad \sigma_{\rho\gamma}^{(0)}|_{\rho=1};$$
$$\sigma_{\rho\rho}^{(j)}|_{\rho=1} = -\sum_{m=0}^{j-1}\left[\Lambda_1^{(j-m)}\sigma_{rr}^{(m)} + \Lambda_2^{(j-m)}\left(\sigma_{\theta\theta}^{(m)} - \sigma_{rr}^{(m)}\right) + \Lambda_3^{(j-m)}\sigma_{r\theta}^{(m)}\right]|_{\rho=1}; \tag{9}$$
$$\sigma_{\rho\gamma}^{(j)}|_{\rho=1} = -\sum_{m=0}^{j-1}\left[\Lambda_4^{(j-m)}\sigma_{r\theta}^{(m)} + \frac{1}{2}\Lambda_3^{(j-m)}\left(\sigma_{\theta\theta}^{(m)} - \sigma_{rr}^{(m)}\right)\right]|_{\rho=1}(j \geq 1).$$

In the considered axisymmetric case, the components $\sigma_{rr}^{(j)}$, $\sigma_{\theta\theta}^{(j)}$, $\sigma_{\alpha\alpha}^{(j)}$, $\sigma_{r\theta}^{(j)}$ have the form

$$\sigma_{rr}^{(j)} = \frac{1}{r_0^2}\sum_{n=0}^{\infty}\sum_{i=2,4} A_n^{(i,J)}\rho^{v_n^{(i)}-\frac{3}{2}}\gamma_n^{(i)}P_n(\mu);$$
$$\begin{pmatrix}\sigma_{\theta\theta}^{(j)}\\ \sigma_{\alpha\alpha}^{(j)}\end{pmatrix} = \frac{1}{r_0^2}\sum_{n=0}^{\infty}\sum_{i=2,4} A_n^{(i,j)}\rho^{v_n^{(i)}-\frac{3}{2}}\left[\begin{pmatrix}\eta_n^{(i)}\\ q_n^{(i)}\end{pmatrix}P_n(\mu) \pm (c_{11}-c_{12})\mu P_n'(\mu)\right]; \tag{10}$$
$$\sigma_{r\theta}^{(i)} = \frac{1}{r_0^2}\sum_{n=0}^{\infty}\sum_{i=2,4}\rho^{v_n^{(i)}-\frac{3}{2}}\delta_n^{(i)}\frac{dP_n(\mu)}{d\gamma} \qquad \left(P_n' = \frac{dP_n}{d\mu}; \quad \mu = \cos\gamma\right).$$

Here, $P_n(\mu)$ are Legendre polynomials [13]; C_{ij} are elastic constants [14]:

$$\gamma_n^{(i)} = K_n^{(i)}\left[2c_{13} + c_{33}\left(v_n^{(i)} - \frac{1}{2}\right)\right] - n(n+1)c_{13}; \qquad \delta_n^{(i)} = c_{44}\left(K_n^{(i)} + v_n^{(i)} - \frac{3}{2}\right);$$
$$\begin{pmatrix}\eta_n^{(i)}\\ q_n^{(i)}\end{pmatrix} = K_n^{(i)}[c_{11} + c_{12} + c_{13}\left(v_n^{(i)} - \frac{1}{2}\right) - n(n+1)\begin{pmatrix}c_{11}\\ c_{12}\end{pmatrix}; \tag{11}$$
$$K_n^{(i)} = \frac{c_{11}n(n+1) - c_{44}\left(v_n^{(i)2} - \frac{1}{4}\right) - c_{11} + c_{12} + 2c_{44}}{(c_{13}+c_{44})\left(v_n^{(i)} - \frac{1}{2}\right) + c_{11} + c_{12} + 2c_{44}}.$$

Arbitrary constants $A_n^{(i,j)}$ according to (4), (7) are found from the boundary conditions (8) and have the form

$$A_0^{(2,j)} = \frac{c_0^{(j)}}{\gamma_0^{(2)}}; \quad A_n^{(2,j)} = -\frac{c_n^{(j)}\delta_n^{(4)} - d_n^{(j)}\gamma_n^{(4)}}{\delta_n^{(2)}\gamma_n^{(4)} - \delta_n^{(4)}\gamma_n^{(2)}}; \quad A_n^{(4,j)} = -\frac{c_n^{(j)}\delta_n^{(2)} - d_n^{(j)}\gamma_n^{(2)}}{\delta_n^{(2)}\gamma_n^{(4)} - \delta_n^{(4)}\gamma_n^{(2)}}; \tag{12}$$
$$(j = 0 \sim n = 0; \quad j = 1 \sim n = 0,2,4,6; \quad j = 2 \sim n = 0,2,4,6,8,10,12),$$

if $c_n^{(j)}, d_n^{(j)}$ are the coefficients of the expansions into series in terms of Legendre polynomials and their derivatives. They depend both on the shape of the surface and on the solution of the boundary value problem in previous approximations. Numerical calculations were carried out for homogeneous media; their elastic constants are given in Table 1.

Table 1

Values of elastic constants for some materials

Material	v_{12}	v_{13}	E_1/G	E_3/G	E_1/E_3
1	0.300	0.300	2.600	2.600	1.000
2	0.300	0.100	5.000	1.250	4.000
3	0.357	0.253	2.771	3.094	0.896
4	0.365	0.288	2.244	2.712	0.828

The materials were used in [14]; they are close to the elastic constants of some transversely isotropic hexagonal crystals [7, 15]. The values for materials 3, 4 are close to the isotropic case 1.

Stress concentration coefficients [1]

$$k_{ii}^{(2)} = \frac{1}{P}\left(\sigma_{ii}^{*(0)} + \varepsilon\sigma_{ii}^{(1)} + \varepsilon^2\sigma_{ii}^{(2)}\right);$$
$$\left(\sigma_{ii}^{*(0)} = \hat{\sigma}_{ii}^{(0)} + \sigma_{ii}^{(0)};\ \ i = \rho, \gamma, \varphi\right) \qquad (13)$$

for all materials of the considered surface shapes (Fig. 1, 2) if $\rho = 1$ have the following analytical structure:

$$k_{ii}^{(2)} = \sum_{k=0,2,\ldots}^{12} a_{ki} P_k(\mu)\,. \qquad (14)$$

The analytical structure of stress concentration coefficients in powers of the variable ρ при $(\gamma = const)$ significantly depends on the type of material and has the following form:

$$k_{ii}^{(2)} = 1 + \sum_{k=0}^{13} b_{ki}\rho^{-k-3}. \qquad (15)$$

Here b_{ki} and a_{ki} in (13) are known numerical coefficients that depend on the elastic constants c_{ij} of the transversely isotropic medium. In particular, the values

$$k_{\rho\rho}^{(0)} = 1 - \frac{1}{\rho^3};\qquad k_{\gamma\gamma}^{(0)} = k_{\varphi\varphi}^{(0)} = 1 + \frac{1}{2\rho^3};\qquad k_{\rho\gamma}^{(0)} = 0 \qquad (16)$$

correspond to the exact solution of the problem for an isotropic medium with a spherical cavity. Therefore, $k_{\gamma\gamma}^{(0)} = k_{\varphi\varphi}^{(0)} = 1{,}5$.

The nature of the change in the stress concentration coefficients $k_{\gamma\gamma}^{(0)}$ and $k_{\varphi\varphi}^{(2)}$ along the fourth meridional sections of non-canonical cavities in the form of regular hexagons of rotation with rounded corners is shown in Fig. 3 for $\varepsilon = 1/15$ and in Fig. 4 for $\varepsilon =- 1/15$.

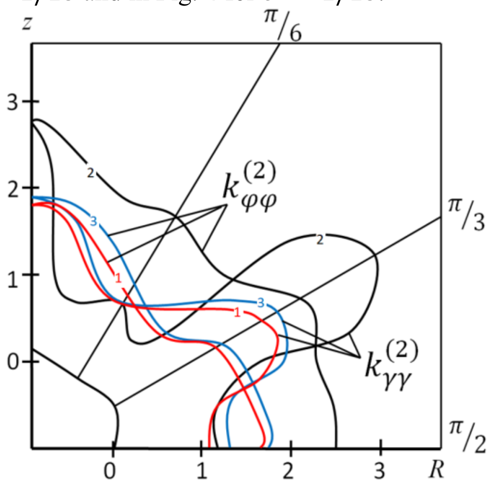

Fig. 3. Stress distribution of the meridional section for $\varepsilon = 1/15$ (1- –, 2 - –, 3 - –)

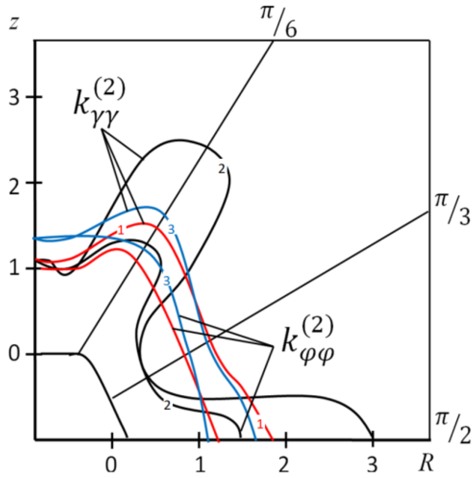

Fig. 4. Stress distribution of the meridional section for $\varepsilon = -1/15$ (1- –, 2 - –, 3 - –)

The change in stress concentration coefficients with distance from the cavity surfaces is shown for $k_{\gamma\gamma}^{(2)}$ in Fig. 5 and $k_{\varphi\varphi}^{(2)}$ in Fig. 6 (materials 1, 2). The local nature of the stress field is also preserved for materials 3, 4.

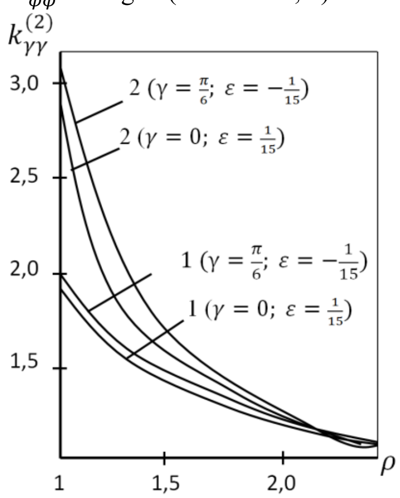

Fig. 5. Distribution of stress concentration coefficients $k_{\gamma\gamma}^{(2)}$ in the vicinity of non-canonical surface

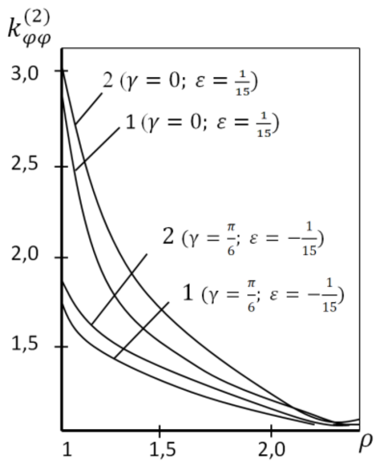

Fig.6. Distribution of stress concentration coefficients $k_{\varphi\varphi}^{(2)}$ in the vicinity of non-canonical surface

For the four materials considered in Table 1, the maximum value of the relative deviation of the values of stress concentration coefficients in the vicinity of non-canonical surfaces from the corresponding values of the basic stress state, which is taken as 100%, does not exceed 5.8% at ρ=2 and 1.7% at ρ=3.

Remark: if a transversely isotropic medium with a non-canonical cavity is under an internal pressure of intensity p applied to its surface ρ=1, then the boundary conditions have a form similar to (8), i.e.

$$\sigma_{\rho\rho}^{(j)}|_{\rho=1} = -p; \qquad \sigma_{\rho\gamma}^{(j)}|_{\rho=1} = 0. \tag{17}$$

In this case, the numerical values for the stress concentration coefficients $k_{\gamma\gamma}^{(2)}$ and $k_{\varphi\varphi}^{(2)}$ are obtained from the results of solving the given problem, if all the numerical values shown in Figures 3-6 are reduced by one.

Conclusions

1. The stressed state of a transversely isotropic medium with non-canonical cavities in the form of regular hexagons of rotation has a pronounced local character. 2. The values of normal stresses in the vicinity of non-canonical cavities depend on the anisotropy of the material. This is especially noticeable if the elastic constants differ significantly from the corresponding values in the isotropic case (illustrated on the example of material 2). 3. In the case of the remark made, the locality of the stress field is preserved. In this case, the results obtained with an error of up to 2% correspond to thick-walled shells formed by coordinate surfaces ρ=1, $\rho = \rho_0 \geq 3$ and are under internal pressure.

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