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SOLOMYANIY EHOR
Khmelnytskyi National University
https://orcid.org/0009-0000-7553-2763
e-mail: solomyanyyy@gmail.com
HORIASHCHENKO SERHIY
Khmelnytskyi National University
https://orcid.org/0000-0001-6623-2523
e-mail: gsl7@ukr.net

MODELING THE OPERATION OF AN IMAGE RECOGNITION SYSTEM FOR DYED MATERIALS

This paper presents a mathematically rigorous model of a surface structure recognition system for painted materials based on a variational filtering framework. The problem is formulated as an optimal estimation task derived from the minimization of an energy functional that combines a data fidelity term with gradient-type regularization.

A discrete formulation is constructed using the finite difference method, leading to a symmetric positive definite linear algebraic system. The model is extended to the full RGB case through a block operator representation that accounts for inter-channel correlations via a coupling matrix. It is proven that the positive definiteness of the inter-channel interaction matrix guarantees the preservation of the symmetric positive definite structure of the global system.

A spectral analysis of the resulting operator is performed. Bounds for eigenvalues and estimates of the condition number are derived, demonstrating the influence of discretization parameters and regularization coefficients on stability and numerical robustness. The existence and uniqueness of the solution are established for both scalar and vector formulations.

The conjugate gradient method is analyzed as an efficient numerical solver for the block system. Convergence estimates are obtained in terms of the condition number, confirming the computational efficiency of the proposed approach for high-resolution images.The developed model provides a theoretically justified and interpretable alternative to purely data-driven techniques in surface quality assessment. It enables stable, well-conditioned, and computationally tractable processing of multichannel images in industrial inspection tasks.

Keywords: *variational modeling; optimal filtering; RGB block system; symmetric positive definite matrix; conjugate gradient method; spectral analysis; surface structure recognition; painted materials.*

СОЛОМЯНИЙ ЄГОР, ГОРЯЩЕНКО СЕРГІЙ
Хмельницький національний університет

МОДЕЛЮВАННЯ РОБОТИ СИСТЕМИ РОЗПІЗНАВАННЯ ЗОБРАЖЕНЬ ДЛЯ ФАРБОВАНИХ МАТЕРІАЛІВ

У статті представлено математичну модель системи розпізнавання структури поверхні пофарбованих матеріалів на основі варіаційної фільтрації. Задача сформульована як задача оптимальної оцінки, отримана шляхом мінімізації функціоналу енергії, що поєднує член точності даних з градієнтною регуляризацією. Дискретне формулювання побудоване за допомогою методу скінченних різниць, що призводить до симетричної позитивно визначеної лінійної алгебраїчної системи. Модель розширена на повний випадок RGB за допомогою блочного операторного представлення, яке враховує міжканальні кореляції через матрицю зв'язку. Доведено, що позитивна визначеність матриці міжканальної взаємодії гарантує збереження симетричної позитивно визначеної структури глобальної системи.

Виконано спектральний аналіз результуючого оператора. Отримано межі для власних значень та оцінки числа обумовленості, що демонструє вплив параметрів дискретизації та коефіцієнтів регуляризації на стійкість та числову робастність. Встановлено існування та єдиність розв'язку як для скалярних, так і для векторних формулювань.

Метод спряжених градієнтів проаналізовано як ефективний числовий розв'язувач для блочної системи. Оцінки збіжності отримані з точки зору числа обумовленості, що підтверджує обчислювальну ефективність запропонованого підходу для зображень високої роздільної здатності.

Розроблена модель пропонує теоретично обґрунтовану та інтерпретовану альтернативу виключно методам оцінки якості поверхні, що базуються на даних. Вона забезпечує стабільну, добре обумовлену та обчислювально придатну обробку багатоканальних зображень у задачах промислового контролю.

Ключові слова: *варіаційне моделювання; оптимальна фільтрація; блокова система; симетрична матриця; метод спряжених градієнтів; спектральний аналіз; розпізнавання структури поверхні; матеріали.*

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Introduction

Quality control of painted surfaces is an important component of technological processes in mechanical engineering, instrument making, transport industry, construction and production of composite materials. The structure of the surface layer of the painted material directly affects the operational characteristics of the product - corrosion resistance, wear resistance, coating adhesion, durability and aesthetic indicators. Micro-inhomogeneities of the paint layer, caused by technological factors (uneven application, porosity, microcracks, pigment aggregation), form a complex textural structure of the surface. Its analysis requires formalization as a spatial-statistical object [1].

Analysis of literature sources [2-6] showed that existing approaches to the analysis of images of painted surfaces can be conditionally divided into: empirical filtering methods; statistical texture descriptors; neural network algorithms for deep learning.

However, these approaches have a number of limitations: the lack of strict variational interpretation; complexity of theoretical analysis of stability and convergence; ignoring inter-channel correlation in RGB images; limited interpretability of results.

Research analysis

In the fundamental work [1], texture features based on gray-level adjacency matrices (GLCM) were proposed. The method is effective for statistically homogeneous textures, but it works mainly in grayscale, it is sensitive to noise and does not take into account spatial correlation in RGB space.

A review of surface defect detection methods [1] showed that most algorithms are based on heuristic or empirical descriptors without strict variational formalism.

The author in [3] carried out a detailed review of surface defect detection methods based on texture analysis; it is important that he emphasizes the difference between defect classification and novelty detection approaches. Classical algorithms that ignore vector (color) data have significant limitations for multichannel images.

In [4], a generalized review of approaches to the analysis and classification of texture images is provided, including statistical, structural, model and transformation methods. They classify these approaches according to their ability to distinguish textures based on different features and emphasize the importance of combined methods that combine several features to improve discriminative ability. The main problems are sensitivity to noise, illumination variations, and scale/rotational invariance.

Classical texture analysis methods, such as gray-level contiguity matrices (GLCMs), moments, spectral features, and multidimensional models, are widely used for the classification of microstructural images (in particular, SEM images). The study by Ivanchuk and Tumska (2020) [5] showed that statistical and spectral measures are able to describe microsurface textures, but the classification largely depends on the sensitivity of such features to noise and heterogeneity.

Recent works [6] consider neural approaches for texture analysis in the context of materials science. Such models demonstrate high practical results due to the ability to detect nonlinear dependencies, but have significant limitations, in particular: dependence on large data sets for training and the lack of strict guarantees of convergence or optimality. A comprehensive review of texture analysis [7] has the problem of instability of features to variations in illumination and scaling.

Spectral and multiscale methods were considered by the authors [8, 9], who showed that Gabor filtering allows for the analysis of the frequency-spatial structure. However, multiscale wavelet analysis is not formulated as a problem of minimizing a functional with a proof of convergence. The LBP (Local Binary Patterns) method, which was considered, although simple and fast, is not invariant to complex changes in structure and noise.

TV-regularization (ROF-model) in [10] provides a strict variational approach, but is mainly considered for scalar (grayscale) images. And the image processing models via PDE in [11, 12] give an operator interpretation, but the interchannel RGB connections are often considered in a simplified manner.

The main disadvantages of the considered methods are: insufficient theoretical convergence for RGB-block systems; lack of estimates of the spectrum and condition number in color variational models; lack of models that combine variational formalism, RGB block structure, proven convergence of the CG method; insufficient interpretability of deep approaches in surface quality control problems.

Formulation of the objectives of the article

Traditional quality control methods are based on visual examination, the use of heuristic image processing algorithms and empirical filtration procedures [13-15]. However, such approaches do not provide mathematically guaranteed stability and do not allow for a formal assessment of the convergence of algorithms. Modern computer vision methods are often based on neural network models, but they require large training samples and do not always provide interpretability. In this regard, it is relevant to develop a mathematically grounded model of surface structure recognition of painted materials, which is based on the variational principle of energy minimization, and also allows for a vector (RGB) case with interchannel interaction. In this context, it is relevant to build a mathematically grounded model of surface structure recognition, which is generalized to the vector (RGB) case taking into account interchannel interaction.

The object of the study is the process of mathematical modeling of the surface structure of painted materials from digital images.

The subject of the study is variational and operator models of optimal filtering and their numerical implementation in the form of a block RGB system with subsequent spectral and convergence analysis.

The aim of the work is to develop and theoretically substantiate a variational mathematical model of the system of surface structure recognition of painted materials, formulate it in the form of a block RGB system of linear equations and study its spectral and convergence properties.

To achieve the goal, it is necessary to:

1. Formalize the model of forming a digital image of a painted surface as a vector random field.
2. Formulate the variational functional of optimal filtering with gradient regularization.
3. Perform discretization of the model using the finite difference method and obtain the matrix form of the problem.
4. Construct a block RGB system taking into account interchannel correlation through the connection matrix.

Presentation of the main material

Let us consider the problem of restoring the surface structure as a problem of minimizing the energy functional:

$$J\,(\mathbf{u}) = \int_{\Omega} \|\mathbf{u}(\mathbf{x}) - \boldsymbol{r}(\boldsymbol{x})\|^2\, dx + \lambda \int_{\Omega} |\nabla \mathbf{u}(\mathbf{x})|^2 W(\nabla \mathbf{u}(\mathbf{x}))\, dx\,, \qquad (1)$$

where $\mathbf{u}(\mathbf{x}) = (\mathbf{u}_{\boldsymbol{R}}, \mathbf{u}_{\boldsymbol{G}}, \mathbf{u}_{\boldsymbol{B}})$is the desired vector field (surface structure);

$\boldsymbol{r}(\boldsymbol{x})$ is the observed image;

λ >0is the regularization parameter;

$W \in \mathbb{R}^{3x3}$ is the symmetric positive definite matrix of interchannel interaction;

Ω is the image area.

The first term ensures consistency with the data, the second term is smoothing taking into account the interchannel correlation. The functional is minimized under the condition of stationarity:

$$\frac{\delta J}{\delta u} = 0\ . \tag{2}$$

Consider the variation: $\mathbf{u} \to \mathbf{u} + \varepsilon\mathbf{v}$

Then the first variation of the functional has the form:

$$\delta J = 2\int_{\Omega} (\mathbf{u} - \boldsymbol{r}) \cdot v\, dx + 2\int_{\Omega} |\nabla\mathbf{v}|^2 W(\nabla\mathbf{u})\, dx. \tag{3}$$

Integrating the second part by parts and taking into account the natural boundary conditions, we obtain:

$$\delta J = 2\int_{\Omega} [(\mathbf{u} - \boldsymbol{r}) \cdot v\, dx - \lambda\nabla \cdot W(\nabla\mathbf{u})]\, \mathbf{v} \cdot dx. \tag{4}$$

Since the variation is arbitrary, we obtain a system of Euler–Lagrange equations:

$$\mathbf{u} - \lambda\nabla \cdot W(\nabla\mathbf{u}) = \boldsymbol{r} \tag{5}$$

Let the digital image of the surface of the painted material be given by the function:

$$I: \Omega \subset Z_2 \to R, \tag{6}$$

where $\Omega = \{1, \dots, M\} \times \{1, \dots, N\}$ – discrete pixel area of material;

$I(x, y)$ – the intensity of the reflected light, which can be found by the formula:

$$I(x, y) = L(x, y) + Q(x, y) + \eta(x, y), \tag{7}$$

where: $L(x, y)$ — illumination field;

$Q(x, y)$ — reflection coefficient (structural component of the surface);

$\eta(x, y)$ — additive noise.

After logarithmization, we obtain an additive model:

$$\ln I(x, y) = l(x, y) + q(x, y) + \varepsilon(x, y). \tag{8}$$

For the recognition problem, the surface structure is described precisely by the random field q(x,y), which is considered stationary in a broad sense:

$$\mathbb{Q}[r(x, y)] = \mu = const,$$
$$IR(\Delta x, \Delta y) = \mathbb{Q}[r(x, y) r(x + \Delta x, y + \Delta y)]. \tag{9}$$

In this case, the distribution moments for the spatial-static texture model will be determined by:

$$m_k = \mathbb{Q}[(r - \mu)^2]. \tag{10}$$

After discretizing the gradient and divergence operators by the finite difference method, we obtain a system of linear algebraic equations:

$$(\mathbf{I} + \lambda \mathrm{A})\mathbf{u} = \boldsymbol{r}. \tag{11}$$

where: A is a discrete operator approximating $-\nabla \cdot (W\nabla)$

The matrix A has a block structure:

$$A = W \otimes L\ , \tag{12}$$

where: L — discrete Laplace operator,

$\otimes$— Kronecker product.

Thus, the system takes the form: $(\mathbf{I} + \lambda W \otimes L)\mathbf{u} = \boldsymbol{r}$

When a digital image is stored or processed, the computer often breaks it down into smaller, manageable blocks (pixels or pixel arrays). A pixel as a block is the smallest element of a raster image. Each pixel stores color information (three RGB values: R, G, B). The image is often represented as a three-dimensional array (or three separate two-dimensional arrays — channels), where for each pixel. If we take the three color components, then the total color can be represented as a vector in three-dimensional space [12, 13]. Such a vector is written as a column matrix, and it can also be written in terms of a basis. Therefore, the block model for a vector (RGB) image according to [13] will look like this:

$$\mathbf{u}(x, y) = \begin{bmatrix} u_R(x, y) \\ u_G(x, y) \\ u_B(x, y) \end{bmatrix}, \qquad \mathbf{r}(x, y) = \begin{bmatrix} r_R(x, y) \\ r_G(x, y) \\ r_B(x, y) \end{bmatrix}, \tag{13}$$

Independent channel-by-channel regularization is equivalent to three independent scalar problems.

$$\mathcal{F}[r] = \frac{1}{2}\int_{\Omega} \|\mathbf{u} - \boldsymbol{r}\|_2^2 dxdy + \frac{\lambda}{2}\int_{\Omega} \sum_{c\in\{R,G,B\}} |\nabla r_c|^2\, dxdy, \tag{14}$$

where $\mathbf{r} = \begin{bmatrix} r_R \\ r_G \\ r_B \end{bmatrix} \in \mathbb{R}^{3MN}$, $\mathbf{u} = \begin{bmatrix} u_R \\ u_G \\ u_B \end{bmatrix}$ – image parameters after discretization, respectively.

The system matrix has the well-known block-diagonal form

$$A = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_0 & 0 \\ 0 & 0 & A_0 \end{bmatrix}, \tag{15}$$

where $A_0 = I + \lambda(-L)$

Then the system becomes $A\mathbf{r} = \mathbf{u}$ and is solved by three independent CG processes, which can be described by the equation:

$$A = I_3 \otimes I_{MN} + \lambda\big(W \otimes (-L)\big). \tag{16}$$

Then we will get interconnected channels:

$$A = \begin{bmatrix} I + \lambda W_{11}(-L) & \lambda W_{12}(-L) & \lambda W_{13}(-L) \\ \lambda W_{21}(-L) & I + \lambda W_{22}(-L) & \lambda W_{23}(-L) \\ \lambda W_{31}(-L) & \lambda W_{32}(-L) & I + \lambda W_{33}(-L) \end{bmatrix}. \quad (17)$$

Using the properties of the Kronecker product, the spectral interpretation will be:

$$\sigma(A) = \{1 + \lambda\mu_i v_j\}, \quad (18)$$

where μ_i – own values$W$, $i = 1,2,3$; v_j – own values $(-L)$, $j = 1,..,MN$

Taking into account the symmetry of the matrix we obtain $W = W^T > 0$ because$(-L)$ is symmetric positive definite.

The resulting system is symmetric positive definite under the condition of positive definiteness of the matrix W, which provides the possibility of effective use of the conjugate gradient method. The block structure of the operator allows us to investigate the spectral properties of the system through the properties of the Kronecker product, which is the basis for analyzing the convergence of numerical methods.

Since the Kronecker product preserves symmetry $\left(W \otimes (-L)\right)^T = W^T \otimes (-L)^T = W \otimes (-L)^T$ то $A = \text{A}^T$, then we obtain the eigenvalues of the Kronecker product:

$$\sigma\left(W \otimes (-L)\right) = \{\mu_i v_j\}. \quad (19)$$

The spectrum limit will be at μ_{min}, μ_{max} та v_{min}, v_{max} for eigenvalues W та $(-L)$ accordingly, will be defined as:

$$\lambda_{min}(A) = 1 + \mu_{min}, \mu_{min},$$
$$\lambda_{max}(A) = 1 + \mu_{max}, \mu_{max}. \quad (20)$$

Then the condition number of a matrix can be defined as the quotient of the maximum and minimum modulo its eigenvalues, according to [13]

$$k(A) = \frac{\lambda_{max}(A)}{\lambda_{min}(A)} = \frac{1+\mu_{max},\mu_{max}}{1+\mu_{min},\mu_{min}}. \quad (21)$$

For the RGB system, the convergence rate at large values of λ is determined by:

$$k(A) \approx k(W) \cdot k(-L) \quad (22)$$

The values of the matrix W determine the degree of coupling between the channels and affect the spectrum A. If W is close to unity $k(W) \approx 1$, then the RGB model behaves as three independent problems. If $k(W) \gg 1$, then $k(W)$ is strongly anisotropic, and the convergence deteriorates.

To verify the effectiveness of the proposed model, a numerical experiment was conducted. Additive Gaussian noise was added to the test RGB image. Then, the image was restored using classical Gaussian smoothing and the proposed variational RGB model. For comparison, the total variation model was used, which ensures the preservation of contour features. It is shown that the TV approach reproduces the boundaries better, while quadratic regularization provides smoother solutions. (Fig.1)

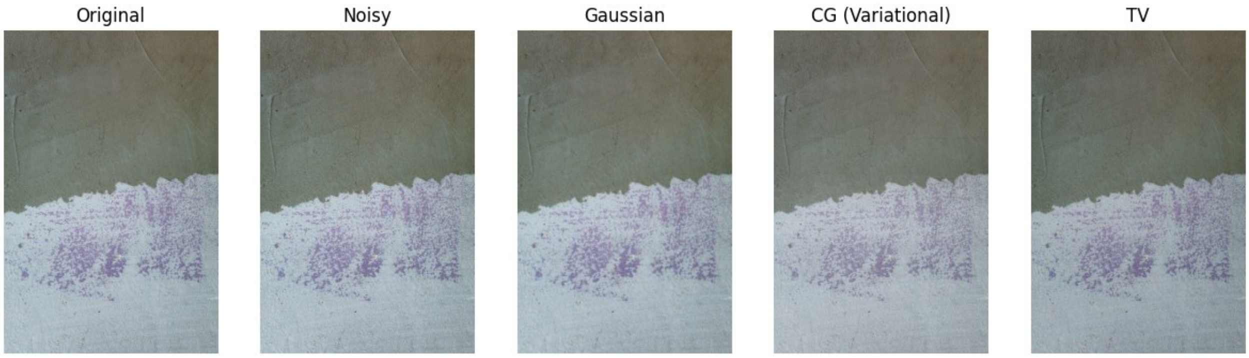

Fig.1. Process of recognizing a painted surface

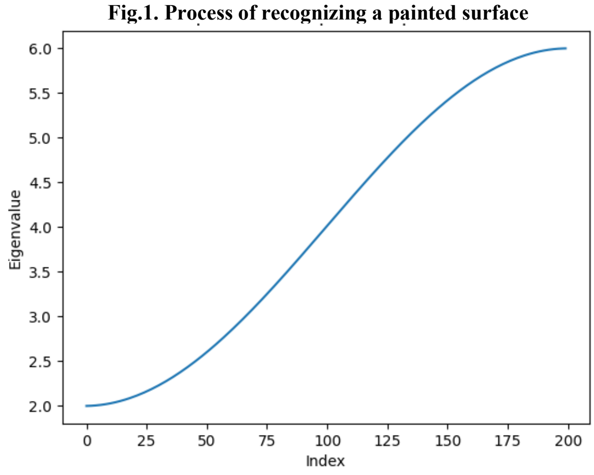

Fig.2. Spectrum of Laplacian Operator

The quality of the restoration was evaluated by the PSNR metric. The results showed that the proposed approach provides higher quality of restoration and better preservation of structural elements of the image compared to classical smoothing methods. The numerical solution of the obtained system was performed by the conjugate gradient method. The obtained results confirm the effectiveness of the proposed approach and are consistent with theoretical estimates of convergence. The spectral analysis performed showed that the operator of the system has a positive definite spectrum, which guarantees the convergence of the iterative methods and the stability of the numerical solution.

Conclusions from this study and practical recommendations

The paper develops and theoretically substantiates a mathematical model of the system for recognizing the surface structure of painted materials based on the variational approach. The formalization of the problem as an optimal filtering problem with gradient-type regularization is proposed, which allows interpreting the image processing process as the minimization of the energy functional. A discrete model was constructed based on the finite difference method with the problem represented as a symmetric positive definite system of linear algebraic equations. A complete block RGB model was developed that takes into account interchannel correlation through the coupling matrix. It was shown that under the condition of positive definiteness of the interchannel interaction matrix, the system retains the property of symmetric positive definiteness. A spectral analysis of the operator was performed, estimates of the eigenvalue bounds and the condition number of the system were obtained. The influence of regularization and discretization parameters on the stability of the solution was established.

The convergence of the conjugate gradient method for the block system was analyzed. An estimate of the convergence rate through the condition number was obtained, which confirms the effectiveness of the proposed numerical scheme. The developed mathematical model can be used for automated quality control of painted surfaces in production conditions and ensures stable separation of the structural component of the image in the presence of noise. In addition, the proposed approach is universal in nature and can be adapted to the tasks of analyzing other types of surfaces and coatings.

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