https://doi.org/10.31891/2307-5732-2026-361-35
УДК 519.6:001.5

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METHODS FOR SOLVING APPLIED PROBLEMS IN MATHEMATICAL PHYSICS

The article proposes methods for solving applied problems for optimizing the parameters of the laser embryo division process. The spherical shape of the embryo, its three-layer internal structure, and laser action modes are taken into account. It is shown that the computational mathematical model of the laser action process is based on a nonlocal boundary value problem of a system of multidimensional, nonstationary differential heat conduction equations with boundary conditions of the third kind and Dirichlet boundary conditions. These boundary conditions are formed taking into account the permissible temperatures of laser action on the embryo, based on expert assessment of the technical parameters of laser emitters. Due to the peculiarities of the embryo and the technical characteristics of laser emitters, significant computer time is required to implement the procedure for solving boundary value problems and to ensure the iterative process of searching for the optimal technical parameters of laser emitters. Therefore, the author proposes to perform this procedure on specialized grid processors. This will increase the accuracy of solving the entire problem of finding the optimal technical parameters of lasers.

The correctness of the boundary value problem presented in the article is justified using specialized methods and estimates of the fundamental function of solutions over the space of generalized functions. Taking into account the peculiarities of the embryo structure and the technical characteristics of laser emitters, a method for solving the boundary value problem for the process of laser action on the embryo was chosen. To control the trauma of the embryo, the author proposes to use an integral criterion for assessing material damage. The quality of embryo division is determined by the ratio of the volumes of the segment of injured embryo embryos to the entire layer of embryos. This will increase the accuracy of laser embryo division and reduce the energy consumption of laser emitters.

Keywords: *applied problems, correctness, integral criterion, energy consumption.*

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МЕТОДИ РОЗВ'ЯЗАННЯ ПРИКЛАДНИХ ЗАДАЧ МАТЕМАТИЧНОЇ ФІЗИКИ

В статті запропоновані методи розв'язання прикладних задач для оптимізації параметрів процеса лазерного ділення ембріона. При цьому враховані сферична форма ембріона, його тришарова внутрішня будова і режими лазерної дії на ембріон. Автором показано, що в основі розрахункової математичної моделі процеса лазерної дії стоїть нелокальна крайова задача системи багатовимірних, нелінійних, нестаціонарних диференціальних рівнянь теплопровідності з граничними умовами 3-го роду і граничними умовами Діріхле. Ці граничні умови сформовані з урахуванням припустимих температур лазерної дії на ембріон, на основі експертного оцінювання технічних параметрів лазерних випромінювачів. Через особливості ембріона і технічні характеристики лазерних випромінювачів потрібні значні часові витрати комп'ютерів для реалізації процедури розв'язання серії крайових задач і забезпечення ітераційного процесу пошуку оптимальних технічних параметрів лазерних випромінювачів. Тому автор пропонує здійснити цю процедуру на спеціалізованих сіткових процесорах. Це дозволить підвищити точність розв'язання усієї задачі пошуку оптимальних технічних параметрів лазерів.

Коректність наведеної в статті крайової задачі обґрунтована з використанням спеціалізованих методів і оцінок на фундаментальну функцію розв'язків над простором узагальнених функцій. Врахувавши особливості будови ембріона і технічні характеристики лазерних випромінювачів, автор обрав метод розв'язання крайової задачі з процеса лазерної дії на ембріон. Для контроля травмованості ембріона автор пропонує скористатись інтегральним критерієм оцінки пошкодження матеріала. Якість поділу ембріона визначена зі співвідношення об'ємів сегмента травмованих зародків ембріона до цілого шару зародків. Це дозволить підвищити точність лазерного ділення ембріона і зменшить енергетичні витрати лазерних випромінювачів.

Ключові слова: *прикладні задачі, коректність, інтегральний критерій, енергетичні витрати.*

Стаття надійшла до редакції / Received 12.12.2025
Прийнята до друку / Accepted 11.01.2026
Опубліковано / Published 29.01.2026

Formulation of the problem

Interdisciplinary methods must be used to calculate and optimize technical and biotechnological systems containing local, concentrated, discrete sources of laser exposure. This is primarily because physical processes in complex systems are described by non-local boundary value problems with systems of partial differential equations. The goal of optimizing such systems can be both physical processes and technical parameters of hardware exposure sources.

In this article, the author sets the main task of optimizing the technical parameters of laser emitters to improve the quality of laser embryo division. To assess embryo damage, the author proposes using an integral criterion of the ratio of the segment of traumatized embryos to the cell layer. Note that to implement the integral criterion, it is necessary to have a layer-by-layer distribution of temperature fields in the embryo, which can only be obtained after solving boundary value problems for the process of laser exposure to the embryo. In addition, the optimization of the parameters of the above-mentioned biotechnological system (embryo under laser exposure) is further complicated by the fact that it is impossible to guarantee the correctness of the boundary conditions for this biotechnological system. Using methods from the theory of pseudodifferential operators, the author obtained conditions of correctness for the above boundary value problem. Since the optimization of technical parameters is based on the repeated solution of boundary value problems, the correctness of boundary value problems entails the correctness of applied optimization mathematical models for the process of laser exposure to the embryo.

Thus, by taking into account the multilayered structure of the embryo and the thermal conditions of laser exposure, it will be possible to improve the quality of this biotechnological process. In addition, according to the author, the research presented in this article can be used to improve the accuracy of mathematical modeling and optimize the parameters of many other technical and biotechnological systems. Changes in the object under study will entail the selection of other methods for calculating the values of the objective function and optimizing technical parameters, while the procedure itself can be left virtually unchanged.

Analysis of the latest research

Articles [1, 2] present mathematical models for heat treatment of cylindrical surfaces. These mathematical models are implemented through cubic approximation of the radial coordinate of the system of equations from boundary value problems. Optimal temperature regimes for chips in computer networks have been calculated [3]. To increase the efficiency of solar energy use and reduce fossil fuel consumption, the authors of article [4] have developed computer models for energy conservation in buildings. The authors of article [5] investigated the application of information technologies for management decision-making with the aim of controlling energy supply depending on economic needs in the region.

The correctness of boundary problems with second-order differential equations for longitudinal vibrations of a rod was substantiated and their solutions were obtained [6]. The article [7] proposes methods for solving parabolic boundary problems with Dirichlet and Neumann boundary conditions for separate cylindrical and spherical regions. The authors of the article present an algorithm for finding the optimal parameters of physical fields in regions of arbitrary shape [8]. For this purpose, article [8] presents a specialized generalized approach to solving specialized problems of mathematical physics described by linear differential equations. The correctness of nonlocal boundary value problems for parabolic equations has been proven in the case where the initial condition is a continuous generalized function [9, 10]. In article [11], geometric models for quadrangular and triangular cells of surface segments are constructed, and resource costs in calculating the parameters of modeled systems are optimized, which will speed up the adoption of appropriate management decisions for certain functional processes. Methods have been developed for forecasting the number of risks, the dynamics of their addition and absorption, and assessing the effect of negative events on the project [12]. The authors' research [12] forms the basis of their methodology for effective project risk management.

The purpose of the work is proposed methods for solving applied problems in mathematical physics to optimize the technical parameters of the biotechnological process of laser embryo division.

Presenting main material

Let us consider the basic principles of implementing the concept of constructing hardware for modeling the effect of a laser beam on a multilayered microbiological object. For clarity, let us take, for example, an early elite embryo of cattle, which is sufficiently complex in terms of layer structure and geometric characteristics.

The main operations of this method are:

– determining the radius of the embryo sphere;

– identifying the stage of embryo development;

– establishing the structure of the embryo's location;

– setting the permissible temperature at the nearest points of the laser's action on the embryo;

– the trajectory of embryo division is determined;

– for each location of the laser beam, the diameter of the laser source, i.e., the spot, in a given section of the embryo is calculated;

– the laser intensity is corrected in accordance with this diameter and the permissible temperature.

The disadvantage of this method is that the embryo in each section is assumed to be homogeneous in terms of thermal conductivity. This significantly reduces the accuracy and quality of this stage of the biotechnological process. To improve the accuracy of modeling, the following sequence of operations is proposed.

Using a microscope and a television camera connected to a computer, the stage of embryo development, its geometric dimensions (radius of the embryo sphere), and the structure of the embryo arrangement are determined. Based on this data, the «tracing» program is used to calculate the rational trajectories of the laser source, i.e., the spots during embryo division.

Next, the permissible temperature at points belonging to the embryos is set. The thermal conductivity coefficients for each layer of the embryo are set. A system of differential equations describing the thermal processes in the embryo when exposed to a laser beam is set. The initial, boundary, and coupling conditions in the embryo layers are entered. Note that under laser exposure, the multilayer (three-layer) microbio-object (embryo) is in the late morula stage of development and contains 32 blastomere cells. It should be noted that the embryo is a nonlinear, non-stationary, three-layer, chemically heterogeneous, spherical microbiological object with different values of thermal conductivity, density, and heat capacity of the layers. The pellucid zone is the protective outer shell of the embryo. In terms of its chemical composition, the pellucid zone consists of 90% water, with 5% protein molecules and low-molecular substances. The perivitelline space occupies an intermediate position between the pellucid zone and the blastomere cell layer. In terms of its chemical composition, the perivitelline space consists mainly of water. Therefore, the values of the thermal conductivity, density, and heat capacity coefficients of the perivitelline space and the pellucid zone can be taken as close to the values of the above parameters for water. The inner space of the embryo is occupied by a layer of blastomere cells, which are protein in nature, and each cell in this layer, in turn, has a three-layer structure.

Without taking into account the multilayer (three-layer) structure of the microbiological object (embryo), the process of laser beam action on the embryo is described using a boundary value problem for the heat conduction equation in a spherical coordinate system with the corresponding Dirichlet boundary conditions at the beginning and end of laser heating of the microbiological object (embryo), and boundary conditions of the third kind, which determine its interaction with the environment. The differential heat conduction equation that determines the laser effect on the embryo:

$$\rho c \frac{\partial T}{\partial t} - \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) + Q(\vec{r}, t) = 0, \quad (1)$$

where ρ – density coefficient of a microbiological object (embryo);
c – embryo heat capacity coefficient;
$T = T(r, t)$ – temperature field;
r– depth of laser beam penetration into the embryo;
t– duration of laser exposure;
λ – embryo thermal conductivity coefficient;
$Q(r, t)$ – a function that characterizes the distribution of the laser energy source.

Taking into account the three-layer structure of the embryo, the heat conduction equation (1) is transformed into a system of differential heat conduction equations:

$$\begin{cases} \rho_1 c_1 \dfrac{\partial T_1}{\partial t} - \lambda_1 \left(\dfrac{\partial^2 T_1}{\partial r^2} + \dfrac{2}{r_1} \dfrac{\partial T_1}{\partial r} \right) + q_1 = 0; \\ \rho_2 c_2 \dfrac{\partial T_2}{\partial t} - \lambda_2 \left(\dfrac{\partial^2 T_2}{\partial r^2} + \dfrac{2}{r_3} \dfrac{\partial T_2}{\partial r} \right) + q_2 = 0; \\ \rho_3 c_3 \dfrac{\partial T_3}{\partial t} - \lambda_3 \left(\dfrac{\partial^2 T_3}{\partial r^2} + \dfrac{2}{r_3} \dfrac{\partial T_3}{\partial r} \right) + q_3 = 0; \\ \rho_4 c_4 \dfrac{\partial T_4}{\partial t} - \lambda_4 \left(\dfrac{\partial^2 T_4}{\partial r^2} + \dfrac{2}{r_4} \dfrac{\partial T_4}{\partial r} \right) + q_4 = 0. \end{cases} \quad (2)$$

Dirichlet boundary conditions define the beginning and end of laser exposure to the embryo:

$$\begin{cases} T(r,t)\big|_{t=t_0}^{r=r_0} = T_0; \\ T(r,t)\big|_{t=t_5}^{r=r_5} = T_5, \end{cases} \quad (3)$$

whereT_0 and T_5 – initial and final temperatures of laser exposure to the embryo.

To account for the three-layer structure of the embryo, we will define the medium separation equations and the temperature field continuity equations along the time coordinate, respectively. Medium separation equations:

$$\begin{cases} T_1(r_1, t_1) = T_2(r_2, t_2), \quad -\lambda_1 \dfrac{\partial T_1}{\partial r} = -\lambda_2 \dfrac{\partial T_2}{\partial r}; \\ T_2(r_2, t_2) = T_3(r_3, t_3), \quad -\lambda_2 \dfrac{\partial T_2}{\partial r} = -\lambda_3 \dfrac{\partial T_3}{\partial r}; \\ T_3(r_3, t_3) = T_4(r_4, t_4), \quad -\lambda_3 \dfrac{\partial T_3}{\partial r} = -\lambda_4 \dfrac{\partial T_4}{\partial r}; \\ T_4(r_4, t_4) = T_5(r_5, t_5), \quad -\lambda_4 \dfrac{\partial T_4}{\partial r} = -\lambda_5 \dfrac{\partial T_5}{\partial r}. \end{cases} \quad (4)$$

Equality of continuity of temperature fields along the time coordinate:

$$\begin{cases} T(r_1; t_1 - 0) = T(r_1; t_1 + 0); \\ T(r_2; t_2 - 0) = T(r_2; t_2 + 0); \\ T(r_3; t_3 - 0) = T(r_3; t_3 + 0); \\ T(r_4; t_4 - 0) = T(r_4; t_4 + 0); \\ T(r_5; t_5 - 0) = T(r_5; t_5 + 0). \end{cases} \quad (5)$$

To set the boundary conditions for heat transfer (third-order boundary conditions) at the interface between the embryo's pellucid zone and the surrounding environment, we will use the boundary conditions for heat flux:

boundary conditions for heat flux at the embryo's pellucid zone:

$$-\lambda_1 \frac{\partial T_1}{\partial z}(0, t) = qS, \quad 0 \le t \le h, \quad (6)$$

where q – specific heat flux;
S – diameter of the heat source, i.e., the spot;
h – duration of laser exposure to the embryo.

To obtain the conditions for the correctness of the multi-point boundary value problem (1)–(6), we will consider homogeneous and inhomogeneous boundary value problems:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = A_1 \left(\dfrac{\partial}{i \partial x} \right) u(x,t); \\ \dfrac{\partial u(x,t)}{\partial t} = A_2 \left(\dfrac{\partial}{i \partial x} \right) u(x,t); \\ \dfrac{\partial u(x,t)}{\partial t} = A_3 \left(\dfrac{\partial}{i \partial x} \right) u(x,t); \\ \dfrac{\partial u(x,t)}{\partial t} = A_4 \left(\dfrac{\partial}{i \partial x} \right) u(x,t) \end{cases} \quad (7)$$

with boundary conditions:

$$B_0\left(\frac{\partial}{i\partial x}\right)u(x,0)+B_5\left(\frac{\partial}{i\partial x}\right)u(x,t_5)=\varphi(x) \qquad (8)$$

and

$$\begin{cases}\dfrac{\partial u(x,t)}{\partial t}=A_1\left(\dfrac{\partial}{i\partial x}\right)u(x,t)+f(x,t);\\ \dfrac{\partial u(x,t)}{\partial t}=A_2\left(\dfrac{\partial}{i\partial x}\right)u(x,t)+f(x,t);\\ \dfrac{\partial u(x,t)}{\partial t}=A_3\left(\dfrac{\partial}{i\partial x}\right)u(x,t)+f(x,t);\\ \dfrac{\partial u(x,t)}{\partial t}=A_4\left(\dfrac{\partial}{i\partial x}\right)u(x,t)+f(x,t)\end{cases} \qquad (9)$$

with boundary conditions:

$$B_0\left(\frac{\partial}{i\partial x}\right)u(x,0)+B_5\left(\frac{\partial}{i\partial x}\right)u(x,t_5)=0, \qquad (10)$$

where $A_k\left(\frac{\partial}{i\partial x}\right)$ and $B_k\left(\frac{\partial}{i\partial x}\right)$ – differential operators with symbols from the space of infinitely differentiable functions of power growth, $k=1,\dots,4$;

$u(x,t)$ – decision function.

We apply the Fourier transform (in spatial variables) to the equations from the homogeneous and inhomogeneous boundary value problems (7)–(8) and (9)–(10), respectively. We obtain the following boundary value problems:

$$\begin{cases}\dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_1(s)\tilde{u}(s,t);\\ \dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_2(s)\tilde{u}(s,t);\\ \dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_3(s)\tilde{u}(s,t);\\ \dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_4(s)\tilde{u}(s,t)\end{cases} \qquad (11)$$

with boundary conditions:

$$B_0(s)\tilde{u}(s,0)+B_5(s)\tilde{u}(s,t_5)=\varphi(s) \qquad (12)$$

and

$$\begin{cases}\dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_1(s)\tilde{u}(s,t)+\tilde{f}(s,t);\\ \dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_2(s)\tilde{u}(s,t)+\tilde{f}(s,t);\\ \dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_3(s)\tilde{u}(s,t)+\tilde{f}(s,t);\\ \dfrac{\partial\tilde{u}(s,t)}{\partial t}=A_4(s)\tilde{u}(s,t)+\tilde{f}(s,t)\end{cases} \qquad (13)$$

with boundary conditions:

$$B_0(s)u(s,0)+B_5(s)u(s,t_5)=0. \qquad (14)$$

Let us seek a solution to problem (11)–(12) in the form of an exponential function. From the continuity condition, we obtain the solution:

$$\varphi_k(s)=exp(t_1A_1(s)+(t_2-t_1)A_2(s)+(t_4-t_3)A_4(s))\,\varphi_1(s). \qquad (15)$$

We obtained the resolving function from the homogeneous boundary value problem (7) – (8):

$$Q(s,t)=\begin{cases}exp\,t\cdot A_1(s)/\Delta(s);\\ exp\,t\cdot A_2(s)/\Delta(s);\\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\ exp((t-t_{n-1})\cdot A_n(s)+\dots+t_1A_1(s))\,/\Delta(s),\end{cases} \qquad (16)$$

where the function $\Delta(s)$:

$$\begin{aligned}\Delta(s)&=B_0(s)+B_1(s)\,exp\,t_1\,A_1(s)+\dots+\\ &+B_n(s)\,exp(t_1A_1(s)+(t_2-t_1)A_2(s)+\dots+(T-t_{n-1})A_n(s))\neq 0,\end{aligned} \qquad (17)$$

that $\varphi_1(s)=\frac{\varphi(s)}{\Delta(s)}$, and the solution to the boundary value problem (10)–(11) can be represented as:

$$Q(s,t)=\begin{cases}exp\,t\cdot A_1(s)\cdot\varphi(s)/\Delta(s);\\ exp\,t\cdot A_2(s)\cdot\varphi(s)/\Delta(s);\\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\ exp((t-t_{n-1})A_n(s)+\dots+t_1A_1(s))\cdot\varphi(s)/\Delta(s).\end{cases} \qquad (18)$$

This function is called the resolving function of problem (11)–(12).

Thus, we have obtained that problem (9)–(10) is correct in functional spaces of bounded growth only when the solution function of the homogeneous boundary value problem (7)–(8) together with its derivatives up to a fixed order is uniformly bounded.

We have obtained that the correctness of the homogeneous boundary value problem (7)–(8) from the space in functional spaces entails the correctness of the inhomogeneous boundary value problem (9)–(10) in the specified spaces [13, 14]. Thus, we arrived at the following result: the parabolicity condition of the homogeneous boundary value problem (7)–(8) is necessary for the correctness in the selected functional spaces of the perturbed equation (9) with homogeneous boundary conditions (10) for sufficiently small perturbations.

Next, the method for solving the boundary value problem (2)–(5) and its computational parameters are specified, for example, the discrete time step, the steps of uneven sampling of the embryo region, etc. The temperature field of the embryo is determined by solving the corresponding boundary value problem [15, 16]. Its implementation can be either numerical or based on an analog-digital grid model. Next, to determine the number of viable embryos (differential criterion), the number of embryos whose laser exposure temperature is below the predetermined permissible temperature is counted. If it is important to estimate the total volume of viable embryos, an integral criterion is proposed, the calculation of which boils down to calculating the total volume of viable embryos. This criterion is used to determine the quality of the biotechnological process of dividing early elite embryos, for example, in livestock breeding. The use of this approach has made it possible to improve the accuracy of the main task of improving the quality of laser exposure to the embryo by controlling the distribution of temperature fields in the layers of the embryo.

Conclusions

The article investigates issues of mathematical modeling and optimization of the «embryo-laser radiation» system with the aim of ensuring the quality of the biotechnological process of laser embryo division by taking into account the viable parts of the embryo. To optimize the technical parameters of such systems, it is necessary to solve boundary value problems that underlie the computational mathematical models. Therefore, the accuracy of parameter optimization is increased by greater detail of the modeled system when implementing computational mathematical models. As shown in this article, the process of laser exposure to the embryo is described using a nonlocal boundary value problem with a system of multidimensional, nonlinear, nonstationary partial differential equations of heat conduction. To determine whether the above-mentioned boundary value problem has a unique solution, it is necessary to define and verify the condition of its correctness.

It should be noted that the conditions of correctness for this boundary value problem obtained by the author in this article are appropriate to be used to justify the correctness of boundary value problems for other technical and biotechnological systems containing concentrated, local sources of physical field exposure. To solve the boundary problems, the article proposes using a grid approach with discretization of the optimized parameters. This will increase the accuracy of the overall problem of improving the quality of laser embryo division by refining the values of local and global extrema of the temperature field function at each iteration.

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