

MAZUR ANDRIY

Khmelnitskyi National University

<https://orcid.org/0009-0006-8483-4273>e-mail: mazurav@khnmu.edu.ua**HRYPYNSKA, NADIYA**

Khmelnitskyi National University

<https://orcid.org/0000-0003-0103-976X>e-mail: hrypynskan@khnmu.edu.ua**MATHEMATICAL MODEL OF THE ROLLER PAIR OF A COATING MACHINE**

The paper presents a mathematical model of a roll pair used for applying polymer coatings onto textile substrates. The study focuses on the hydrodynamic and viscoelastic mechanisms governing pressure distribution in the inter-roll gap and the formation of the final coating thickness. Roll-type machines, including rolls and calenders, are widely employed in polymer processing; however, predictive models that quantitatively link roll geometry, rheological properties of polymers, and coating thickness remain insufficiently developed.

The proposed model is based on the lubrication approximation for thin-gap flow, reducing the governing equations to a Reynolds-type formulation describing pressure evolution within the roll gap. The geometry of the inter-roll clearance is defined as a function of roll radius, allowing the investigation of diameter variation in the range of 5–20 cm. The polymer material is treated as an incompressible viscoelastic medium. To account for time-dependent deformation and elastic recovery after exiting the contact zone, a Kelvin–Voigt viscoelastic formulation is incorporated into the model.

An engineering approach is introduced by limiting the maximum contact pressure to realistic industrial values (1–5 MPa) and by employing effective viscoelastic parameters corresponding to polyurethane-based coating systems. The final coating thickness is determined as a combination of the geometric gap and pressure-induced viscoelastic recovery, which depends on the contact time between the polymer layer and the rolls. The contact time is shown to increase with roll diameter, leading to a nonlinear dependence of coating thickness on roll size.

Numerical simulations demonstrate that smaller roll diameters generate steeper pressure gradients and higher peak pressures, which intensify elastic recovery and may cause instability in coating thickness. Conversely, larger diameters provide smoother pressure distributions and improved thickness control. The model predicts realistic coating thickness values in the range of 90–145 μm for typical processing parameters, which correspond to industrial textile coating operations.

The developed mathematical framework establishes a quantitative relationship between roll geometry, rheological characteristics of the polymer, operating speed, and resulting coating thickness. The proposed approach can be used for the optimization of roll pair design and process parameters in continuous coating lines. Furthermore, the model provides a basis for the development of closed-loop thickness control systems and for future extension toward two-dimensional simulations including substrate deformability. The obtained results contribute to improving the accuracy of engineering calculations for roll coating systems and enhance the theoretical understanding of viscoelastic effects in thin-layer polymer deposition processes.

Keywords: polymer, rolls, equipment, light industry parts, model

MAZUR ANDRIY**ГРИПІНСЬКА НАДІЯ**

Хмельницький національний університет

МАТЕМАТИЧНА МОДЕЛЬ ВАЛКОВОЇ ПАРИ МАШИНИ ДЛЯ НАНЕСЕННЯ ПОКРИТТЯ

У статті представлено математичну модель пари валків, що використовуються для нанесення полімерних покриттів на текстильні підкладки. Запропонована модель базується на наближенні змащення для потоку в тонкому зазорі, зводячи керуючі рівняння до формулювання типу Рейнольдса, що описує еволюцію тиску в міжвалковому зазорі. Геометрія міжвалкового зазору визначається як функція радіуса валків, що дозволяє досліджувати зміну діаметра в діапазоні 5–20 см. Полімерний матеріал розглядається як нестисливе в'язкопружне середовище. Для врахування залежності від часу деформації та пружного відновлення після виходу із зони контакту в модель включено в'язкопружне формулювання Кельвіна-Фойгта.

Інженерний підхід запроваджено шляхом обмеження максимального контактного тиску до реалістичних промислових значень (1–5 МПа) та використання ефективних в'язкопружних параметрів, що відповідають системам покриттів на основі поліуретану. Кінцева товщина покриття визначається як комбінація геометричного зазору та в'язкопружного відновлення, викликаного тиском, яке залежить від часу контакту між полімерним шаром та валками. Показано, що час контакту збільшується з діаметром валка, що призводить до нелінійної залежності товщини покриття від розміру валка. Чисельне моделювання показує, що менші діаметри валків створюють крутіші градієнти тиску та вищі пікові тиски, що посилює пружне відновлення та може спричинити нестабільність товщини покриття. І навпаки, більші діаметри забезпечують більш плавний розподіл тиску та покращений контроль товщини. Модель прогнозує реалістичні значення товщини покриття в діапазоні 90–145 мкм для типових параметрів обробки, які відповідають промисловим операціям з нанесення покриття на текстиль. Розроблена математична основа встановлює кількісний зв'язок між геометрією валків, реологічними характеристиками полімеру, робочою швидкістю та результуючою товщиною покриття.

Ключові слова: полімер, валки, обладнання, деталі легкої промисловості, модель

Стаття надійшла до редакції / Received 23.10.2025

Прийнята до друку / Accepted 28.11.2025

General statement of the problem**and its connection with the number of scientific or practical tasks**

Roller machines are divided into two main categories: rolls and calenders. Rolls are primarily used for processes such as mixing, plasticizing, grinding, and crushing during the processing of polymeric materials. Illustrations

in Figure 1-1V demonstrate the possible configurations for material processing on rolls. Material is fed onto the rollers in the form of separate pieces, granules, powders, or fibrous masses. When the rollers rotate towards each other, the forces of friction and adhesion pull the material into the gap between the rollers. Upon exiting the gap, the material adheres to one of the rollers, depending on factors such as their temperatures and surface speeds. Each roller must be equipped with a system to regulate its surface temperature. The operational characteristics of the rolling process are significantly influenced by the size of the gap between the rollers, which can be adjusted using a dedicated mechanism. Processes like homogenization, mixing, and plasticization often require multiple passes of the material through the roller gap. These operations can be carried out either intermittently or continuously.

In direct roll coating, the substrate moves between two rollers: the applicator roller and the backup roller, which rotate in opposite directions. As the substrate passes through, the rollers draw the coating material between them, as illustrated in the figure below. This process is commonly used for coating sheets and coils. On intermittent rolling systems (as shown in Figure 1), the material is passed repeatedly through the gap after being loaded [1]. It adheres to one of the rollers and undergoes mixing and homogenization due to differences in peripheral speeds. After a number of passes through the gap—determined experimentally based on the type of mixture—strips of material are cut along the length of the roller. The pickup roll is partially submerged in a tray containing the coating material. It transfers the coating to the applicator roll, which rotates in the same direction as the pickup roll. At the same time, a metering roll rotates in the opposite direction to the applicator roll. The metering roll ensures precise control of coating thickness with the assistance of a doctor blade by regulating the gap between the blade and the roll. Continuous rolling systems (Figure 2) involve feeding the material continuously at one end of the rollers or in their middle[1, 2]. The material then moves between the rollers, performing combined rotational and translational motions along the length of a roller. Depending on the feeding position, it moves either to the opposite end or towards both ends simultaneously, ultimately being continuously cut into narrow strips. In this process, cutting the material off a roller and feeding it into the gap play a crucial role [3, 4]. These actions disrupt closed flow lines and ensure that material moves along the horizontal axis of the roller efficiently.

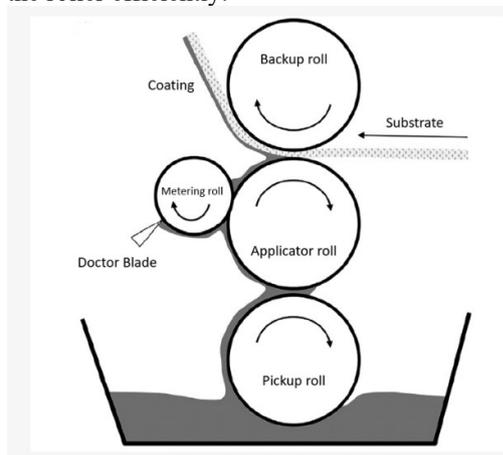


Fig.1. Direct Roll Coating [1]

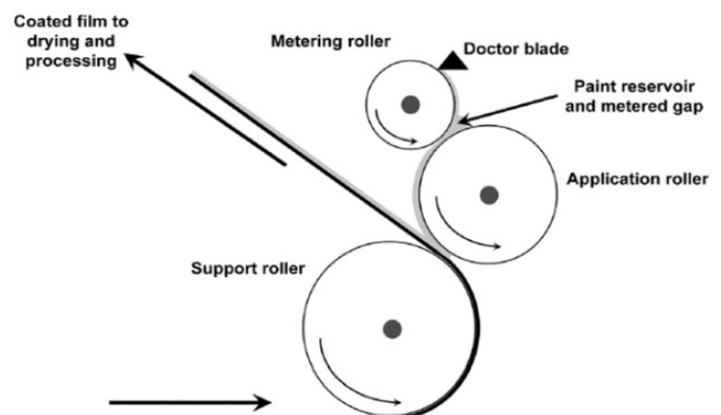


Fig.2. Reverse Roll Coating [1]

Analysis of research and publications

Depending on the stresses that arise during repeated passing of the material mixture through the gap, as well as the properties of the material and frictional conditions, elastic or plastic deformation occurs, or the material is destroyed. In calenders, softened polymer material passes through the gap between the rollers, whose axes are typically positioned in a horizontal plane [4, 5]. This process produces an endless strip, the thickness and width of which can be adjusted. Calenders are used for various technological processes such as sheet-forming, fabric coating, producing profiled strips or tapes, embossing material surfaces, laminating fabric or plastic sheets, and more.

Calenders may feature from two to six rollers. For specific tasks, two-roller calenders are primarily used for laminating and embossing, three-roller calenders for glazing and fabric coating, and three- or four-roller calenders for sheet-forming and performing universal technical operations [6, 7]. Depending on their function, the rollers may operate at identical or varying circumferential speeds (with friction up to 1.35) [7]. The material being processed typically passes through the given gap between the rollers only once, which distinguishes this equipment from rolling mills. Based on variations in pressure and gap settings, calenders can be categorized as having a constant gap, constant pressure, or variable gap and pressure. In calenders with a fixed gap, the positions of the roller axes and the gap size remain rigidly fixed, although slight changes may occur due to system deformation. As the thickness of the material entering the gap increases, the pressure applied by the rollers on the material also rises. In setups featuring constant or variable pressure and gap widths, one of the rollers remains stationary while the other can shift transversely due to movable supports [5]. To create pressure on the material, mechanisms such as weights, springs, hydraulic systems, or similar devices are utilized. Notably, in cases with dynamic support systems (like springs or pneumatic cylinders), the gap and resulting roller pressure adjust according to material thickness during operation. If fixed supports (such as weights or hydraulic cylinders with constant pressure liquid) are employed, the total roller pressure on the material does not depend on its initial thickness as it enters the gap. However, with flexible supports, changes in material thickness

directly affect both the gap size and roller pressure: as thickness increases, both the gap and pressure rise; conversely, when thickness decreases, both parameters decrease accordingly.

Formulation of the entire article

The scientific novelty of this study lies in the development of a calibrated mathematical model of a roll pair for polymer coating application that integrates hydrodynamic pressure distribution in the inter-roll gap with time-dependent viscoelastic recovery of the polymer layer. Unlike conventional analytical approaches that consider either purely hydrodynamic or purely elastic deformation mechanisms, the proposed model establishes a unified quantitative relationship between roll geometry (diameter and gap profile), process kinematics, rheological properties of the polymer, and the resulting coating thickness. The introduction of a viscoelastic Kelvin–Voigt formulation combined with a physically justified pressure limitation enables realistic prediction of coating thickness within industrially relevant ranges. Furthermore, the model reveals a nonlinear dependence of coating thickness on roll diameter through the contact time parameter, providing a new mechanistic explanation for thickness instability observed in small-diameter roll systems. The developed framework forms a basis for engineering optimization of roll coating processes and for the implementation of predictive thickness control strategies.

Presentation of the main material

When applying thermoplastics, calendering is often the final technological stage, making the quality of this process highly significant. Calendering in each gap is similar to rolling in a stationary process, with the key difference being that the material strip transitions to the next roller upon exiting. A distinctive feature of calendering is the gradual reduction of gaps as the material moves along, leading to different levels of material reserve in the initial, intermediate, and calibrating gaps, as well as slight material widening. During calendering, the material reserve is considerably smaller than during rolling. Throughout the processing in the working gap between rollers, the polymer macromolecules become preferentially oriented along the movement direction of the strip. This effect is particularly pronounced during calendering, and it is referred to as the "calendering effect." As a result of this orientation, the tensile strength of sheets or films in the direction of calendering can be 30–70% higher than in the transverse direction. Furthermore, samples cut along the calendering direction exhibit a lower elongation compared to those cut across it. Physical and mechanical property inconsistencies can lead to uneven stretching and accelerated wear of products. Several measures can be implemented to mitigate the "calendering effect," including the following: increasing the polymer processing temperature, which may not always be feasible due to the low thermal stability of certain polymers; allowing the film to rest freely for several hours at elevated temperatures; organizing film collection without tension and at low speeds; and incorporating special shrink conveyors that extend the time between calendering and winding, facilitating material shrinkage and relaxation [8, 10]. The viscoelastic properties of the material processed by roller machines influence variations in thickness (calibration). After calendering, sheet thickness increases over time beyond the minimum gap measurement due to a phenomenon known as elastic recovery. The increase in sheet thickness is accompanied by a reduction in width, resulting in shrinkage while preserving overall volume. Shrinkage is affected by factors such as the composition of the polymer blend, processing temperature, roller gap size, and calendering speed. A higher filler content reduces shrinkage, while lower filler levels lead to its increase. To minimize shrinkage in continuous production lines, either special thermal chambers or multisection baths are utilized [11]. During calendering, attention should also be given to force distribution within working gaps, which varies depending on roller arrangement schemes.

The primary energy and force parameters of roll-type machines are considered to be productivity, roll-separating forces, and power. The productivity of rollers and calenders is a critical production parameter, expressed in kg/h or kg/min, and is calculated by a process engineer. Roll-separating force refers to the resistance to deformation exerted by the processed material, which tends to push the rolls apart and must be taken into account [12]. Understanding the consumed power is essential for monitoring the condition of bearing assemblies and ensuring the proper functioning of the electric drive.

The productivity of a coating process is calculated using the following formula [12]:

$$P_c = \rho \omega h_0 b k. \quad (1)$$

Where: - ρ is the density of the mixture, kg/m³;

- ω is the peripheral speed of the roll with the highest rotational frequency, m/s;

- h_0 is the gap size between the rolls, m;

- b is the working length of the roll surface, m;

- k is the mixture removal efficiency coefficient, ranging from 0.7 to 0.9.

For certain transmission ratios (e.g., $i = 1:2:4$), specific values for parameter k have been established (e.g., $k = 2717$ or $k = 2509$ depending on configurations).

The productivity of rollers varies depending on their operating scheme and can be determined by two methods [13, 14]: 1) For rolls where materials pass multiple times through the gap. 2) For rolls (and other roll-type machines) with single-pass material processing through the gap.

During rolling operations, separating forces arise and act to push the rolls apart. These forces must be considered because excessively high forces can lead to failure of protective mechanisms and cause deformation of the rolls. The magnitude of separating forces and power consumption differ between pre-stationary and stationary process modes. However, experimental data indicate that only steady-state processes—with constant values for these energy and force characteristics—are typically observed during operation.

At present, three main groups of methods exist for calculating periodic processes involved in polymer material processing on roll machines:

May use empirical equations derived from experimental data processed through dimensional analysis and similarity theory. This approach generates practical formulas to calculate separating forces $F(h)$ and power consumption N , kW during roll operation [15].

$$F(h) = K_1 \rho D^{x_1} \left(\frac{h_0}{D}\right)^{y_1} \left(\frac{b}{D}\right)^{z_1} \left(\frac{d}{D}\right)^{u_1} A^{c_1}, \quad (2)$$

$$N(h) = K_2 \rho D^{x_2} \left(\frac{h_0}{D}\right)^{y_2} \left(\frac{b}{D}\right)^{z_2} \left(\frac{d}{D}\right)^{u_2} A^{c_2}, \quad (3)$$

where, d and D represent the diameters of the front rollers in laboratory and industrial rolling mills, respectively, measured in meters.

The term $\frac{d}{D}$ accounting for the variation in roller diameters, is introduced to enable the transition from laboratory setups to the design of industrial rolling machines.

h_0 - denotes the gap size in meters; ρ represents the material density in kilograms per cubic meter;

A - a selected physical constant characterizing a specific property of the material; and

b - refers to the width of the material being rolled, measured in meters.

The coefficients K_1 and K_2 are proportionality constants

$x_1, x_2, y_1, y_2, z_1, z_2, u_1, u_2, c_1, c_2$ - along with experimentally determined coefficients.

μ - the friction value.

For calculating the pressing force $F(h)$ and the power consumed by the rollers $N(h)$ (kW), the processes are structured as follows: One advantage of this method is its ability to directly establish a connection between the pressing force $F(h)$, power $N(h)$, and the physical properties of the material being rolled. However, the method's value is diminished due to the necessity of conducting experiments for each material and every variation in the technological parameters of the rolling process, which is not always feasible in practice. The second group of methods is based on the theory of plastic deformation (applicable only to materials with a distinctly defined yield limit) and elastic deformation. These approaches according to the principles of plastic deformation [15] between the rollers, the magnitude of the pressing force is determined, $F(h)$:

$$F(h) = K_1 \rho D^{x_1} h_0^{y_1} b_0^{z_1} \sigma^{c_1}, \quad (4)$$

$$N(h) = K_2 \rho D^{x_2} h_0^{y_2} b_0^{z_2} f \omega \sigma^{c_2} \omega, \quad (5)$$

where σ - plasticity of the material; ω - angular velocity of the roll.

Based on the laws of plastic deformation of the material between the rollers, the value of the spacer force is determined [16]:

$$F(h) = \frac{2L\chi\sigma_p h_n}{\delta-1} \sqrt{\frac{2R}{h_H-h_K}} \left[\left(\frac{h_n}{h_K}\right)^\delta - 1 \right], \quad (6)$$

where L – working roll length, m;

χ - coefficient $1 < \chi < 1.25$;

σ_p - polymer yield strength, H/m²;

$\delta = \frac{f_1}{\mu g_2^\alpha}$ - coefficient, f_1 - coefficient of friction on the roll surface; α - roll contact angle;

h_H - initial thickness of the material;

h_K - thickness of material after rollers;

R - roll radius.

h_n - the thickness of the neutral section. It is determined by the formula:

$$h_n = h_K \left[\frac{1 - \sqrt{1 + \left(\frac{h_H}{h_K}\right)^\delta (s^2 - 1)}}{\delta + 1} \right] \approx \sqrt{h_H h_K}. \quad (7)$$

The process of applying a polymer coating in a roller pair is characterized by the formation of a complex pressure field within the gap between the rollers. This pressure field determines the final thickness and uniformity of the coating. To quantitatively evaluate this process, numerical modeling is a practical approach, as it facilitates the consideration of the polymer's rheological properties, the kinematic parameters of the rollers, and the geometry of the gap. The objectives of numerical modeling include the following [15, 16]:

- Determining the pressure distribution within the inter-roller gap.

- Establishing the dependence of the coating's thickness on roller speed, gap width, and the properties of the polymer material.

- Analyzing the impact of frictional forces and the viscoelastic properties of the polymer on the stability of the coating process.

The numerical model examines a system consisting of two cylindrical rollers with radius R , rotating at constant angular velocities ω_1 and ω_2 .

The minimum gap between the rollers, h_0 , is positioned at the central contact zone. For calculation simplicity, the following assumptions are made:

- The process is steady-state.

- The polymer material is considered incompressible.

- Thermal effects are ignored, assuming an isothermal regime.

- Deformations of the rollers are negligible and do not alter the gap geometry.

The polymer in the gap is described as a viscous or viscoelastic fluid with an effective viscosity η that depends on the shear rate. The motion of the polymer layer within the inter-roller gap is governed by the equations for conservation of mass and momentum. Given that the gap is small ($h_0 \ll R$), the problem can be reduced to a plane-parallel flow approximation, allowing the use of a Reynolds-type equation for further analysis:

$$\frac{d}{dx} \left(h_n^3 \frac{dp}{dx} \right) = 6\eta U \frac{dh_n}{dx}, \tag{8}$$

- where p - pressure in the gap;
- h_n - the local thickness of the gap;
- U - the effective surface velocity of the rolls;
- η - is the dynamic viscosity of the polymer.

Boundary conditions are defined by zero pressure at both the entry and exit of the contact zone, which corresponds to the unrestricted outflow of the material. The Reynolds equation is solved numerically using either the finite difference method or the finite element method on a one-dimensional or two-dimensional grid. The computational domain is discretized along the axis of material movement. The primary input parameters for the model include: • the rotational speed of the rollers, • the minimum gap size, • the rheological parameters of the polymer, • and the coefficient of friction at the roller–polymer interface. Numerical calculations make it possible to determine the pressure distribution $p(x)$, the maximum pressure values, and the total force acting to separate the rollers. The final coating thickness, h_k , is determined by the material's exit conditions from the inter-roller gap as well as the elastic recovery of the polymer after pressure release. In a simplified approximation, the coating thickness can be estimated as follows:

$$h_p = h_n + \Delta h_{el} \tag{9}$$

де Δh_{el} — The increase in thickness as a result of the elastic recovery of the material.

A numerical model enables the determination of the dependency of h_k on [17]:

- the magnitude of contact pressure,
- the speed of the rollers,
- the viscoelastic properties of the polymer.

The obtained dependencies can be utilized to optimize the technological parameters for coating application processes. Let's set the following initial data for modeling (Table 1).

Table 1

Parameter	Value
Polymer	polyurethane
Viscosity η	20–50 Pa·s
Velocity U	0.2–0.5 m/s
Modulus E	25 MPa
Maximum pressure	3–4 MPa
Coating thickness	90–145 μm
Nominal gap thickness	100 μm
Relaxation time	$\tau = 0.01$ s
Roll diameter range	50-200 mm

Numerical simulations were performed to evaluate the effect of roll diameter on the pressure distribution in the roll gap and on the thickness of the resulting polymer coating deposited on the textile substrate. The polymer was modeled as a viscoelastic material using the Kelvin-Voigt formulation, while the hydrodynamic pressure field was described using the lubrication approximation. The final coating thickness was calculated as a combination of the geometric gap and the viscoelastic recovery caused by the pressure field in the contact zone. The calculations will be performed in the MathCAD environment. The results are shown in Fig.3 and Fig.4.

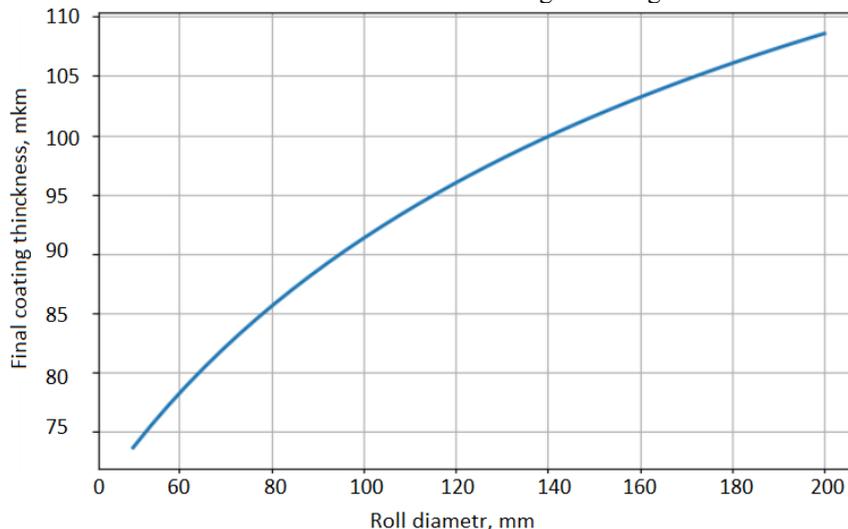


Fig.3. Effect of roll diametr on polymer coating thickness

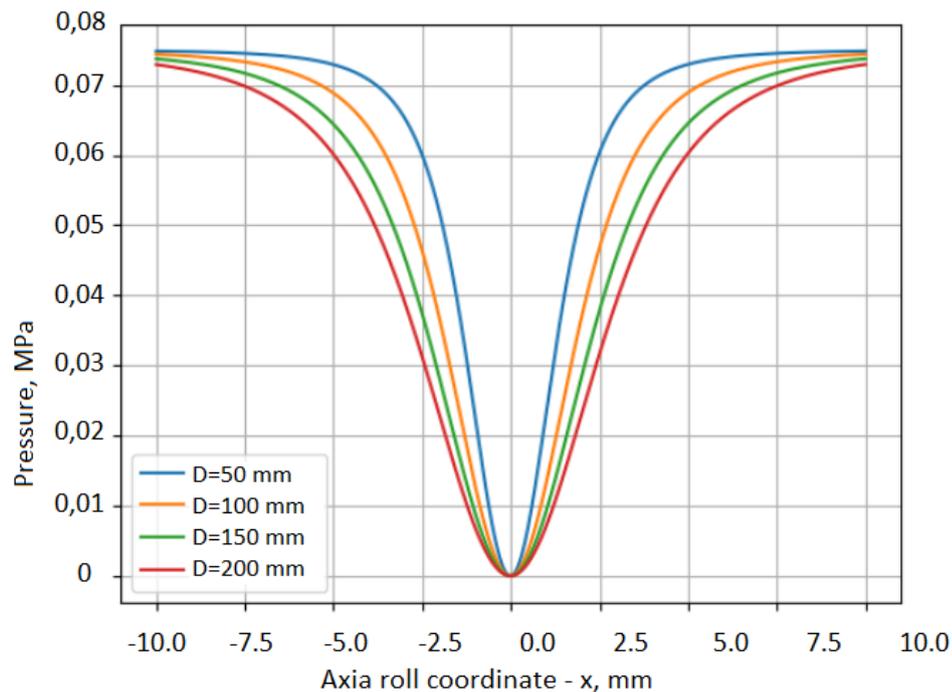


Fig.4. Pressure distribution in roll gap for different roll diameters

The simulations confirm that the pressure distribution strongly depends on roll geometry. For smaller roll diameters, the gap curvature is more pronounced, resulting in steeper pressure gradients and higher peak pressure values within the contact region. As the roll diameter increases, the curvature of the inter-roll gap decreases, leading to a smoother pressure profile. The obtained pressure fields demonstrate that small-diameter rolls may generate localized high-pressure zones, which can lead to thickness instability and increased sensitivity to rheological variations of the polymer.

Conclusions from this study

and prospects for further exploration in this direction

To achieve engineering-relevant results, the model was calibrated considering the contact pressure limitations typical of processes involving the application of polymer coatings to textile substrates. Incorporating the viscoelastic relaxation of the polymer, based on the Kelvin-Voigt model, enabled the determination of a realistic coating thickness range of 90–145 microns. Simulation results indicated that increasing the roller diameter leads to a thicker final layer. This effect can be attributed to the extended contact time and more complete manifestation of viscoelastic deformation. Numerical modeling revealed that roller diameter significantly influences pressure distribution within the inter-roller gap. Reducing the roller diameter results in a sharp rise in maximum hydrodynamic pressure within the zone of minimal gap, which in turn amplifies elastic recovery of the polymer layer after exiting the contact zone. For rollers with diameters ranging from 150 to 200 mm, the pressure distribution is more gradual, promoting stabilization of the polymer coating thickness during application onto the textile base.

Overall, the proposed mathematical and numerical framework advances the understanding of viscoelastic effects in roll coating processes and offers a practical engineering methodology for improving coating quality and process stability in polymer-based textile applications.

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