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IDENTIFICATION OF THE MOTION PARAMETERS OF THE SENSITIVE ELEMENT OF THE LINEAR ACGSELERATION METER

The development of modern high-precision gravimetric and navigation systems requires the improvement of all components of these systems and the wide application of algorithmic methods of processing measurement signals. Possibilities for improving the design and increasing the accuracy of the manufacturing of the constituent elements are practically exhausted at the present time. Therefore, the application of algorithmic methods of increasing the accuracy of gravimetric and navigation systems is a very promising and relevant way. This requires the creation of highly accurate and effective algorithmic methods for processing the output signal of linear acceleration meters, as an important component of gravimetric and navigation systems. The output signal of these meters is formed using a gyroscopic sensitive element that deviates by an angle proportional to the current acceleration. The deflection angle of the sensitive element is measured by the angle sensor. The operation of linear acceleration meters in unfavorable and non-stationary measurement conditions is accompanied by the occurrence of a number of disturbances added to the output signal of the sensitive element of these meters. For example, harmonic interference can be caused by the non-stationary thermal state of the gyroscopic sensitive element and the effect of periodic movements on the frequency of pendulum oscillations of the sensitive element. Therefore, the article takes these features into account when developing identification algorithms for linear acceleration meters with increased metrological characteristics

In the article the algorithmic method of increase of accuracy of accelerometers of linear accelerations is considered. The basis of the given method is the identification of parameters of motion of a sensing element of these accelerometers. The errors of identification because of method of a maximum probability and errors stipulated by a linearization of mathematical model of motion of a sensing element are considered.

Key words: linear acceleration meter, sensitive element, algorithmic methods, accuracy improvement, parameter identification

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ІДЕНТИФІКАЦІЯ ПАРАМЕТРІВ РУХУ ЧУТЛИВОГО ЕЛЕМЕНТА ВИМІРЮВАЧА ЛІНІЙНИХ ПРИСКОРЕНЬ

Розвиток сучасних високоточних гравіметричних і навігаційних систем потребує удосконалення всіх складових елементів цих систем та широкого застосування алгоритмічних методів обробки вимірювальних сигналів. Можливості удосконалення конструкції та підвищення точності виготовлення складових елементів на теперішній час практично вичерпані. Тому дуже перспективним і актуальним шляхом є застосування алгоритмічних методів підвищення точності гравіметричних і навігаційних систем. Це вимагає створення високоточних і ефективних алгоритмічних методів обробки вихідного сигналу вимірювачів лінійних прискорень, як важливої складової частини гравіметричних і навігаційних систем. Вихідний сигнал цих вимірювачів формується за допомогою гіроскопічного чутливого елемента, що відхиляється на кут, пропорційний діючому прискоренню. Кут відхилення чутливого елемента вимірюється датчиком кута. Робота вимірювачів лінійних прискорень в несприятливих та нестаціонарних умовах вимірювань супроводжується виникненням ряду завад, що додаються до вихідного сигналу чутливого елемента цих вимірювачів. Наприклад, гармонійна завада може бути обумовлена нестаціонарним тепловим станом гіроскопічного чутливого елемента та впливом періодичних рухів на частоті маятникових коливань чутливого елемента. Тому у статті враховано ці особливості при розробці алгоритмів ідентифікації для вимірювачів лінійних прискорень з підвищеними метрологічними характеристиками

В статті розглянуто алгоритмічний метод підвищення точності вимірювачів лінійних прискорень. Основою даного методу є ідентифікація параметрів руху чутливого елемента цих вимірювачів. Розглянуто похибки ідентифікації за методом максимальної правдоподібності та похибки, обумовлені лінеаризацією математичної моделі руху чутливого елемента.

Ключові слова: вимірювач лінійних прискорень, чутливий елемент, алгоритмічні методи, підвищення точності, ідентифікація параметрів

Statement of the problem in a general form and its connection with important scientific or practical tasks

The development of modern high-precision gravimetric and navigation systems requires the improvement of all components of these systems and the wide application of algorithmic methods of processing measurement signals. Possibilities for improving the design and increasing the accuracy of the manufacturing of the constituent elements are practically exhausted at the present time. Therefore, the application of algorithmic methods of increasing the accuracy of gravimetric and navigation systems is a very promising and relevant way.

Analysis of recent research and publications

All this requires the creation of highly accurate and effective algorithmic methods for processing the output signal of linear acceleration meters, as an important component of gravimetric and navigation systems [1, 2]. The output signal of these meters is formed with the help of a gyroscopic sensitive element (GSE), which deviates by an angle proportional to the current acceleration. The angle of deviation of the sensitive element (GSE) is measured by the angle sensor (AS).

There are many scientific works devoted to the theoretical foundations and research of optimal algorithms for filtering discrete signals of measuring instruments containing interference [3, 4, 5]. However, these works do not address the issue of identifying the motion parameters of the sensitive element of the linear acceleration meter.

The operation of linear acceleration meters in unfavorable and non-stationary measurement conditions is accompanied by the occurrence of a number of disturbances that are added to the output signal of these meters. In [6], the issue of the effect of harmonic interference, which can be caused by the non-stationary thermal state of the gyroscopic GSE and the effect of periodic movements on the frequency of pendulum oscillations of the GSE, is covered. But the main features are not taken into account when developing identification algorithms for linear acceleration meters with increased metrological characteristics.

The task of optimal filtering is the task of assessing the state of the electric vehicle and identifying its movement parameters, which is formulated in stochastic terms. Therefore, in the future, we will use the term "identification of parameters of GSE movement".

The purpose of this article is to develop an algorithmic method for identifying the parameters of the motion of GSE linear acceleration meters. This method provides an increase in the accuracy of linear acceleration meters in adverse and non-stationary measurement conditions.

Presentation of the main material of the article

We will perform the identification of the movement parameters GSE of the linear acceleration meter based on the processing of data α_i^*, $i=\overline{1,K}$, received from the AS of this vehicle. With

$$\alpha_i^* = \alpha(t_i) + \delta_\alpha(t_i), \quad i=\overline{1,K}, \quad t_i = i\cdot\delta_{\text{ä}}, \quad T_c = K\cdot\delta_{\text{ä}},$$

where $\alpha(t_i)$ – values corresponding to the ideal trajectory of the GSE movement, $\delta_\alpha(t_i)$ – errors of the measured trajectory of the GSE movement, due to the effect of obstacles on the GSE and AS errors, K – the number of readings coming from the AS, $\delta_{\text{ä}}$ – the time interval between readings, T_c – GSE observation time.

The movement of the electric vehicle, observed with the help of AS, can be represented by the sum of the useful component, which is considered constant over the observation interval and which is proportional to the measured acceleration, and the variable component, which is determined by the solution of the differential equation [2, 6]

$$\ddot{\alpha}_{\text{ÇÌ}} + 2\xi_{\text{ÇÌ}}\,\dot{\alpha}_{\text{ÇÌ}} + \omega_0^2 \sin\alpha_{\text{ÇÌ}} = 0, \tag{1}$$

where ω_0 - is the circular frequency of precessional oscillations of GSE, ξ_1 - is the damping parameter.

In the case of small fluctuations of the GSE $\sin(\alpha_{\text{ÇÌ}}) \approx \alpha_{\text{ÇÌ}}$, and the solution of equation (1) has the form $\alpha_{\text{ÇÌ}}(t) = A_{\text{ÇÌ}}\, e^{-\xi_{\text{ÇÌ}} t} \sin(\omega_{\text{ÇÌ}} t + \varphi_{\text{ÇÌ}})$, where $\omega_{\text{ÇÌ}} = \sqrt{\omega_0^2 - \xi_{\text{ÇÌ}}^2}$, $A_{\text{ÇÌ}}, \varphi_{\text{ÇÌ}}$ – are the amplitude and the initial phase of the precessional oscillations of the GSE. If $\xi_1 \to 0$, then the mathematical model of the ideal trajectory of the GSE motion has the form:

$$\alpha(t) = \alpha_{\text{Ï}} + \alpha_{\text{ÇÌ}}(t); \; \alpha_{\text{Ï}} = const; \; \alpha_{\text{ÇÌ}}(t) = \alpha_C \sin\omega_{\text{ÇÌ}} t + \alpha_S \cos\omega_{\text{ÇÌ}} t, \tag{2}$$

where $\alpha_C = A\cos\varphi$, $\alpha_S = A\sin\varphi$. The GSE state vector to be identified is equal to: $Z_\alpha = (\alpha_{\text{Ï}}, \alpha_C, \alpha_S)^T$.

In the general case, the errors $\delta_\alpha(t_i)$ of the measured trajectory of the GSE movement can be correlated, taking into account the presence of deterministic disturbances (harmonic, exponential) and kinematic nonlinearities of the GSE. We will consider the distribution of the error amplitude to be normal, taking into account the influence of many factors that lead to these distortions. All this determines the application of the maximum likelihood method for assessing the state of the GSE.

The maximum likelihood estimate $\hat{Z}_\alpha$ for the state vector Z_α is determined from the equation [3, 7]

$$\frac{d(\ln J(\hat{Z}_\alpha))}{d\hat{Z}_\alpha} = A^T \cdot R_\alpha^{-1}\left(\alpha^* - \alpha\left(\hat{Z}_\alpha, T\right)\right) = 0\,, \tag{3}$$

where

$$A^T = \begin{bmatrix} \frac{\partial\alpha(\hat{Z}_\alpha,t_1)}{\partial\hat{\alpha}_Ï} & \frac{\partial\alpha(\hat{Z}_\alpha,t_2)}{\partial\hat{\alpha}_Ï} & \cdots & \frac{\partial\alpha(\hat{Z}_\alpha,t_K)}{\partial\hat{\alpha}_Ï} \\ \frac{\partial\alpha(\hat{Z}_\alpha,t_1)}{\partial\hat{\alpha}_C} & \frac{\partial\alpha(\hat{Z}_\alpha,t_2)}{\partial\hat{\alpha}_C} & \cdots & \frac{\partial\alpha(\hat{Z}_\alpha,t_K)}{\partial\hat{\alpha}_C} \\ \frac{\partial\alpha(\hat{Z}_\alpha,t_1)}{\partial\hat{\alpha}_S} & \frac{\partial\alpha(\hat{Z}_\alpha,t_2)}{\partial\hat{\alpha}_S} & \cdots & \frac{\partial\alpha(\hat{Z}_\alpha,t_K)}{\partial\hat{\alpha}_S} \end{bmatrix}, \tag{4}$$

$\alpha^* = \left(\alpha_1^*, ..., \alpha_K^*\right)^T$ – the vector of the results of measurements of the trajectory of the GSE movement, $\alpha(\hat{Z}_\alpha, T) = \left(\alpha(\hat{Z}_\alpha, t_1), ..., \alpha(\hat{Z}_\alpha, t_K)\right)^T$ – the vector of the deviation angle values of the GSE calculated for the mathematical model (2) of the ideal movement trajectory of the GSE based on the estimation $\hat{Z}_\alpha$ of the state vector, $T = (t_1, ..., t_K)^T$ – the vector of moments of time at which the measurements of the measured movement trajectory of the GSE were obtained,

$$R_\alpha = \sigma_{ÄÊ}^2 \cdot I_K + R_{×Â}\,; \quad R_\alpha^{-1} = [w_{ji}]\,,\ i,j = \overline{1,K} \tag{5}$$

– correlation matrix of errors of the measured trajectory of the GSE movement, $\sigma_{ÄÊ}^2$ – AS error variance, I_K – unit matrix of size , $K \times K$, $R_{×Â}$ – correlation matrix of errors due to the effect of correlated disturbances on the GSE.

Then

$$\alpha(\hat{Z}_\alpha, t_i) = \hat{\alpha}_Ï + \hat{\alpha}_C \sin(\omega_{Çì}\, t_i) + \hat{\alpha}_S \cos(\omega_{Çì}\, t_i)\,, \tag{6}$$

$$A^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \sin(\omega_{Çì}\,\delta_ä) & \sin(2\omega_{Çì}\,\delta_ä) & \cdots & \sin(K\omega_{Çì}\,\delta_ä) \\ \cos(\omega_{Çì}\,\delta_ä) & \cos(2\omega_{Çì}\,\delta_ä) & \cdots & \cos(K\omega_{Çì}\,\delta_ä) \end{bmatrix}. \tag{7}$$

Let's calculate the maximum likelihood estimate for the GSE state vector based on (3) taking into account (5), (6) and (7):

$$B_\alpha \cdot \hat{Z}_\alpha = C_\alpha\,, \tag{8}$$

where $B_\alpha = \begin{bmatrix} \sum_{i=1}^K w_i & \sum_{i=1}^K w_i \sin(i\omega_{Çì}\,\delta_ä) & \sum_{i=1}^K w_i \cos(i\omega_{Çì}\,\delta_ä) \\ \sum_{i=1}^K w_i \sin(i\omega_{Çì}\,\delta_ä) & \sum_{i=1}^K w_i \sin^2(i\omega_{Çì}\,\delta_ä) & \sum_{i=1}^K w_i \sin(i\omega_{Çì}\,\delta_ä)\cos(i\omega_{Çì}\,\delta_ä) \\ \sum_{i=1}^K w_i \cos(i\omega_{Çì}\,\delta_ä) & \sum_{i=1}^K w_i \sin(i\omega_{Çì}\,\delta_ä)\cos(i\omega_{Çì}\,\delta_ä) & \sum_{i=1}^K w_i \cos^2(i\omega_{Çì}\,\delta_ä) \end{bmatrix}$;

$$C_\alpha = \left[\sum_{i=1}^K w_i\alpha_i^* \quad \sum_{i=1}^K w_i\alpha_i^* \sin(i\omega_{Çì}\,\delta_ä) \quad \sum_{i=1}^K w_i\alpha_i^* \cos(i\omega_{Çì}\,\delta_ä)\right]^T; \quad w_i = \sum_{j=1}^K w_{ji}\,.$$

The solution of the system (8) is related to the estimation of the state vector $\hat{Z}_\alpha$ and is the result of the identification of the parameters of the GSE movement in the linear acceleration meter. This solution can be found by known methods of solving systems of linear algebraic equations and is a linear function relative to the measured readings α_i^*:

$$\hat{\alpha}_Ï = \sum_{i=1}^K \alpha_i^* \cdot l_{\alpha Ï,i}\,, \quad \hat{\alpha}_C = \sum_{i=1}^K \alpha_i^* \cdot l_{\alpha C,i}\,, \quad \hat{\alpha}_S = \sum_{i=1}^K \alpha_i^* \cdot l_{\alpha S,i}\,. \tag{9}$$

The accuracy of estimates of the state and identification of the parameters of the GSE movement, calculated according to (9), is determined by the following errors:

– methodical error of estimation of state vector of GSE according to the method of maximum credibility;

– the error of estimation of the state vector of the GSE, due to the transition from the nonlinear differential equation (1) to the linear mathematical model of the movement of the GSE (2).

Let's consider the methodological error of estimating the state vector of the GSE by the method of maximum likelihood. The maximum likelihood equation (3) includes the vector of the results of measurements of the trajectory of the vehicle movement α^* and the vector of values of the angle of deviation of the vehicle $\alpha(\hat{Z}_\alpha, T)$, calculated for the mathematical model (2) of the ideal movement trajectory of the vehicle based on the estimation $\hat{Z}_\alpha$ of the state vector.

If, according to [7]

$$\alpha^* = \alpha(Z_\alpha, T) + \Delta_\alpha ,$$

$$\alpha(\hat{Z}_\alpha, T) \approx \alpha(Z_\alpha, T) + \frac{\partial \alpha(Z_\alpha, T)}{\partial Z_\alpha} \cdot \Delta_{Z\alpha} ,$$

where $\Delta_{Z\alpha} = \hat{Z}_\alpha - Z_\alpha$ – is the estimation error of the GSE state vector, then on the basis of (3) we obtain:

$$A_\alpha^T R_\alpha^{-1}\left(A_\alpha \Delta_{Z\alpha} - \Delta_\alpha\right) = 0 ,$$

where

$$\Delta_{Z\alpha} = (A_\alpha^T R_\alpha^{-1} A_\alpha)^{-1} \cdot A_\alpha^T R_\alpha^{-1} \Delta_\alpha ,$$

and the correlation matrix of the estimation errors of the GSE state vector

$$\Psi_{\Delta Z\alpha} = E[\Delta_{Z\alpha} \cdot \Delta_{Z\alpha}^T] = (A_\alpha^T R_\alpha^{-1} A_\alpha)^{-1} . \quad (10)$$

Let's calculate the error of estimating the state of the GSE according to formula (10) taking into account formulas (4), (5) and (7):

$$\Psi_{\Delta Z\alpha} =$$

$$= \begin{bmatrix} \sum_{j=1}^{K}\sum_{i=1}^{K} w_{ji} & \sum_{j=1}^{K}\sum_{i=1}^{K}\left(w_{ji}\sin(i\omega_{Çì}\,\delta_{ä})\right) & \sum_{j=1}^{K}\sum_{i=1}^{K}\left(w_{ji}\cos(i\omega_{Çì}\,\delta_{ä})\right) \\ \sum_{j=1}^{K}\left(\sum_{i=1}^{K} w_{ji}\right)\sin(j\omega_{Çì}\,\delta_{ä}) & \sum_{j=1}^{K}\left(\sum_{i=1}^{K} w_{ji}\sin(i\omega_{Çì}\,\delta_{ä})\right)\sin(j\omega_{Çì}\,\delta_{ä}) & \sum_{j=1}^{K}\left(\sum_{i=1}^{K} w_{ji}\cos(i\omega_{Çì}\,\delta_{ä})\right)\sin(j\omega_{Çì}\,\delta_{ä}) \\ \sum_{j=1}^{K}\left(\sum_{i=1}^{K} w_{ji}\right)\cos(j\omega_{Çì}\,\delta_{ä}) & \sum_{j=1}^{K}\left(\sum_{i=1}^{K} w_{ji}\sin(i\omega_{Çì}\,\delta_{ä})\right)\cos(j\omega_{Çì}\,\delta_{ä}) & \sum_{j=1}^{K}\left(\sum_{i=1}^{K} w_{ji}\cos(i\omega_{Çì}\,\delta_{ä})\right)\cos(j\omega_{Çì}\,\delta_{ä}) \end{bmatrix}^{-1} .$$

Let us consider the error of estimation of the state vector of the GSE, caused by the transition from the nonlinear differential equation (1) to the linear mathematical model (2), for the case of correlated errors of the measured trajectory of the GSE movement.

To determine this error, it is necessary to have a refined mathematical model of the movement of the GSE. Such a model can be obtained on the basis of the differential equation (1) by substitution $\sin(\alpha_{Çì}) \approx \alpha_{Çì} - \alpha_{Çì}^3/3!$ [2, 6]. In this case, the refined mathematical model of the GSE movement has the form:

$$\alpha_0(t) = \alpha_{0Ï} + \alpha_{0Çì}(t), \quad \alpha_{0Ï} = const ,$$

$$\alpha_{0Çì}(t) = A_{0Çì}\, e^{-\xi_{Çì} t} \sin(\omega_{0Çì} t + \varphi_{0Çì}) + \frac{A_{0Çì}^3}{192} e^{-\xi_{Çì} t} \sin 3(\omega_{0Çì} t + \varphi_{0Çì}) \approx$$

$$\approx \alpha_{0C} \sin\omega_0 t + \alpha_{0S} \cos\omega_0 t + \frac{A_{0Çì}^3}{192} \sin 3(\omega_{0Çì} t + \varphi_{0Çì}) - \quad , \quad (11)$$

$$- \omega_{0Çì}\, t \frac{A_{0Çì}^3}{16} \cos(\omega_{0Çì} t + \varphi_{0Çì}) - \xi_{Çì}\, t A_{0Çì} \sin(\omega_{0Çì} t + \varphi_{0Çì}).$$

where $A_{0Çì}, \varphi_{0Çì}$ – are the amplitude and the initial phase of the precessional oscillations of the GSE in the refined mathematical model, $\omega_{0Çì} = \omega_0\left(1 - A_{0Çì}^2/16\right)$, $\alpha_{0C} = A_{0Çì} \cos\varphi_{0Çì}$, $\alpha_{0S} = A_{0Çì} \sin\varphi_{0Çì}$, $\xi_{Çì} << \omega_0$, and also only two terms in the expansion $\alpha_{0Çì}(t)$ in the Taylor series according to the parameters $\omega_{0Çì}$ and $\xi_{Çì}$ in the vicinity of the point $(\omega_0, 0)$ are taken into account.

To determine the estimation error of the state vector of the GSE, due to the transition from the nonlinear differential equation (1) to the linear mathematical model (2), we will use the maximum likelihood method. Let's estimate the vector of errors

$$\Delta_{Z\alpha 2} = \hat{Z}_\alpha - \hat{Z}_{0\alpha} = \left(\Delta_{\alpha Ï}, \Delta_{\alpha C}, \Delta_{\alpha S}\right)^T ,$$

where $\hat{Z}_{0\alpha} = \left(\hat{\alpha}_{0Ï}, \hat{\alpha}_{0C}, \hat{\alpha}_{0S}\right)^T$ – is the estimate of the GSE state vector corresponding to the refined mathematical model (11).

At the same time, we will use the probability equation

$$A_0^T R_\alpha^{-1}\left(\alpha\left(\hat{Z}_\alpha, T\right) - \alpha_0\left(\hat{Z}_{0\alpha}, T\right)\right) = 0 , \quad (12)$$

where $\alpha_0(\hat{Z}_{0\alpha}, T) = \left(\alpha_0(\hat{Z}_{0\alpha}, t_1), ..., \alpha_0(\hat{Z}_{0\alpha}, t_K)\right)^T$ – is the vector of values of the deviation angle of the GSE, calculated for the model (11) based on the estimation $\hat{Z}_{0\alpha}$ of the state vector, $A_0^T = \frac{\partial}{\partial \hat{Z}_{0\alpha}}\left(\alpha_0(\hat{Z}_{0\alpha}, T)\right)^T$ – is the matrix of partial derivatives for (11).

Considering that in real conditions of operation of the linear acceleration meter $A_{0Çì} \le 2^\circ \approx 0{,}033\,ðàä$, $A_{0Çì}^3/192 << A_{0Çì}$, the matrix

$$A_0^T \approx \begin{bmatrix} 1 & 1 & \dots & 1 \\ \sin(\omega_{0Çì}\,\delta_{ä}) & \sin(2\omega_{0Çì}\,\delta_{ä}) & \dots & \sin(K\omega_{0Çì}\,\delta_{ä}) \\ \cos(\omega_{0Çì}\,\delta_{ä}) & \cos(2\omega_{0Çì}\,\delta_{ä}) & \dots & \cos(K\omega_{0Çì}\,\delta_{ä}) \end{bmatrix}. \tag{13}$$

In addition, the probability equation (12) is nonlinear with respect to the error in determining the frequency of precessional oscillations $\Delta_\omega = \omega_{Çì} - \omega_{0Çì}$. Therefore, we will perform its linearization by expanding into a Taylor series $\alpha(\hat{Z}_\alpha, T)$ the expression for excluding terms of the second and higher orders:

$$\alpha(\hat{Z}_\alpha, T) \approx \alpha_0(\hat{Z}_{0\alpha}, \omega_0, T) + \frac{\partial \alpha_0(\hat{Z}_{0\alpha}, \omega_0, T)}{\partial \hat{Z}_{0\alpha}} \cdot \Delta_{Z\alpha 2} + \frac{\partial \alpha_0(\hat{Z}_{0\alpha}, \omega_0, T)}{\partial \omega_0} \cdot \Delta_\omega .$$

As a result, for the mathematical model (2) we get:

$$\begin{aligned} \alpha(\hat{Z}_\alpha, t_i) = \hat{\alpha}_{0Ï} + \hat{\alpha}_{0C}\sin(\omega_{0Çì}\,t_i) + \hat{\alpha}_{0S}\cos(\omega_{0Çì}\,t_i) + \Delta_{\alpha Ï} + \Delta_{\alpha C}\sin(\omega_{0Çì}\,t_i) + \\ + \Delta_{\alpha S}\cos(\omega_{0Çì}\,t_i) + \Delta_\omega t_i(\hat{\alpha}_{0C}\cos(\omega_{0Çì}\,t_i) + \hat{\alpha}_{0S}\sin(\omega_{0Çì}\,t_i)). \end{aligned} \tag{14}$$

Substituting (11), (13) and (14) into (12), we get:

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ \sin(\omega_{0Çì}\,\delta_{ä}) & \sin(2\omega_{0Çì}\,\delta_{ä}) & \dots & \sin(K\omega_{0Çì}\,\delta_{ä}) \\ \cos(\omega_{0Çì}\,\delta_{ä}) & \cos(2\omega_{0Çì}\,\delta_{ä}) & \dots & \cos(K\omega_{0Çì}\,\delta_{ä}) \end{bmatrix} \times \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1K} \\ w_{21} & w_{22} & \dots & w_{2K} \\ \dots & \dots & \dots & \dots \\ w_{K1} & w_{K2} & \dots & w_{KK} \end{bmatrix} \times \begin{bmatrix} \hat{\Delta}_{\alpha,1} \\ \hat{\Delta}_{\alpha,2} \\ \dots \\ \hat{\Delta}_{\alpha,K} \end{bmatrix} = 0, \tag{15}$$

where

$$\begin{aligned} \hat{\Delta}_{\alpha,i} = \alpha(\hat{Z}_\alpha, t_i) - \alpha_0(\hat{Z}_{0\alpha}, t_i) = \Delta_{\alpha Ï} + \Delta_{\alpha C}\sin(\omega_{0Çì}\,i\delta_{ä}) + \Delta_{\alpha S}\cos(\omega_{0Çì}\,i\delta_{ä}) - \\ - \frac{\hat{A}_{0Çì}^3}{192}\sin 3(\omega_{0Çì}\,i\delta_{ä} + \hat{\varphi}_{0Çì}) + \frac{\hat{A}_{0Çì}^3}{16}\omega_{0Çì}\,i\delta_{ä}\cos(\omega_{0Çì}\,i\delta_{ä} + \hat{\varphi}_{0Çì}) + \\ + \hat{A}_{0Çì}\,\xi_{Çì}\,i\delta_{ä}\sin(\omega_{0Çì}\,t + \hat{\varphi}_{0Çì}) + \hat{A}_{0Çì}\,\Delta_\omega i\delta_{ä}\cos(\omega_{0Çì}\,i\delta_{ä} + \hat{\varphi}_{0Çì}). \end{aligned}$$

The resulting probability equation (15) can be transformed into a system of three linear equations with respect to the vector of errors in the estimation of the state of the GSE $\Delta_{Z\alpha 2} = (\Delta_{\alpha Ï}, \Delta_{\alpha C}, \Delta_{\alpha S})^T$ and this system can be written in matrix form:

$$B_\Delta \cdot \Delta_{Z\alpha 2} = C_\Delta , \tag{16}$$

where

$$B_\Delta = \begin{bmatrix} \sum_{i=1}^K w_i & \sum_{i=1}^K w_i\sin(i\omega_{0Çì}\,\delta_{ä}) & \sum_{i=1}^K w_i\cos(i\omega_{0Çì}\,\delta_{ä}) \\ \sum_{i=1}^K w_i\sin(i\omega_{0Çì}\,\delta_{ä}) & \sum_{i=1}^K w_i\sin^2(i\omega_{0Çì}\,\delta_{ä}) & \sum_{i=1}^K w_i\sin(i\omega_{0Çì}\,\delta_{ä})\cos(i\omega_{0Çì}\,\delta_{ä}) \\ \sum_{i=1}^K w_i\cos(i\omega_{0Çì}\,\delta_{ä}) & \sum_{i=1}^K w_i\sin(i\omega_{0Çì}\,\delta_{ä})\cos(i\omega_{0Çì}\,\delta_{ä}) & \sum_{i=1}^K w_i\cos^2(i\omega_{0Çì}\,\delta_{ä}) \end{bmatrix};$$

$$C_\Delta = \begin{bmatrix} \sum_{i=1}^K w_i\Delta_{i,i} & \sum_{i=1}^K w_i\Delta_{i,i}\sin(i\omega_{0Çì}\,\delta_{ä}) & \sum_{i=1}^K w_i\Delta_{i,i}\cos(i\omega_{0Çì}\,\delta_{ä}) \end{bmatrix}^T;$$

$$w_i = \sum_{j=1}^K w_{ji} ;$$

$$\begin{aligned} \Delta_{i,i} = \frac{\hat{A}_{0Çì}^3}{192}\sin 3(\omega_{0Çì}\,i\delta_{ä} + \hat{\varphi}_{0Çì}) + \frac{\hat{A}_{0Çì}^3}{16}\omega_{0Çì}\,i\delta_{ä}\cos(\omega_{0Çì}\,i\delta_{ä} + \hat{\varphi}_{0Çì}) + \\ + \hat{A}_{0Çì}\,\xi_{Çì}\,i\delta_{ä}\sin(\omega_{0Çì}\,t + \hat{\varphi}_{0Çì}) + \hat{A}_{0Çì}\,\Delta_\omega i\delta_{ä}\cos(\omega_{0Çì}\,i\delta_{ä} + \hat{\varphi}_{0Çì}). \end{aligned} \tag{17}$$

Expression (17) contains the constituent parts of the error of the estimation of the state vector of the GSE, due to the transition from the nonlinear differential equation (1) to the linear mathematical model (2) and the error of determining the frequency Δ_ω.

The solution of the system (16) relatively $\Delta_{\alpha\Pi}$ allows to determine the error of estimation of the constant component of the GSE movement, which is due to the transition from the nonlinear differential equation (1) to the linear mathematical model (2). This solution was found taking into account the correlation of the errors of the measured trajectory of the GSE movement and is more accurate compared to other known solutions. The specified solution is a combination of matrix B_Δ and vector elements C_Δ :

$$\Delta_{\alpha\Pi} = \frac{B_{\Delta 11}c_{\Delta 1} + B_{\Delta 21}c_{\Delta 2} + B_{\Delta 31}c_{\Delta 3}}{\det(B_\Delta)} , \tag{18}$$

where $B_{\Delta ji}$ – are the algebraic terms of the elements of the $b_{\Delta ji}$ matrix B_{Δ}, $c_{\Delta j}$ – are the elements of the vector C_{Δ}.

Taking into account relations (17) and (18), the expression for calculating the error $\Delta_{\alpha\ddot{I}}$ in general is a nonlinear function that depends on the parameters of the GSE movement, the errors of measuring the angular position of the GSE, and the parameters of the state estimation algorithm of this GSE:

$$\Delta_{\alpha\ddot{I}} = f(\hat{A}_{0\text{Çì}}, \hat{\varphi}_{0\text{Çì}}, R_{\alpha}^{-1}, \omega_{0\text{Çì}}, \Delta_{\omega}, \delta_{\ddot{a}}, K).$$

Algorithmic error compensation is proposed to improve the accuracy of the estimation of the state of the GSE and the accuracy of measurements of linear accelerations $\Delta_{\alpha\Pi}$ (Ukrainian patent for the invention UA 86005 C2 [8]). To do this, it is necessary to perform the following sequence of actions:

1. Preliminarily determine the initial values of the parameters $R_{\alpha}^{-1}, \omega_{0\text{Çì}}, \Delta_{\omega}, \delta_{\ddot{a}}, K$ based on a priori information about the design properties of the linear acceleration meter and the properties of the evaluation algorithm.

2. Obtain the results of measurements of the trajectory of the movement of GSE α_i^*, $i = \overline{1, K}$.

3. Calculate the estimate of the GSE state vector $\hat{Z}_{\alpha} = \left(\hat{\alpha}_{\ddot{I}}, \hat{\alpha}_C, \hat{\alpha}_S\right)^T$ based on the system of equations (8) and (9).

4. Calculate the estimation error $\Delta_{\alpha\ddot{I}}$ of the GSE state vector based on the system of equations (16) and (18). At the same time, the values calculated in point 3 $\hat{A}_{03M}, \hat{\varphi}_{03M}$ can be used to obtain estimates $\hat{\alpha}_C, \hat{\alpha}_S$:

$$\hat{A}_{03M} \approx \sqrt{\hat{\alpha}_C^2 + \hat{\alpha}_S^2}, \quad \hat{\varphi}_{03M} \approx \begin{cases} \arcsin(\hat{\alpha}_S / \hat{A}_{03M}), & \hat{\alpha}_C \geq 0, \\ \pi - \arcsin(\hat{\alpha}_S / \hat{A}_{03M}), & \hat{\alpha}_C \leq 0. \end{cases}$$

5. Calculate the refined value of the constant component of the GSE motion $\hat{\alpha}_{\ddot{I}1}$ and the corresponding value of the linear acceleration $\hat{a}$:

$$\hat{\alpha}_{\ddot{I}1} = \hat{\alpha}_{\ddot{I}} - \Delta_{\alpha\ddot{I}}; \quad \hat{a} = k_{\ddot{I}1} \cdot \hat{\alpha}_{\ddot{I}1},$$

where $\hat{\alpha}_{\ddot{I}}$ is determined by formula (8) or (9), $\Delta_{\alpha\ddot{I}}$ – by formula (18), $k_{\ddot{I}1}$ – the proportionality factor determined on the basis of data on the design of the linear acceleration meter.

Conclusions

An effective way to increase the accuracy of linear acceleration meters is to identify the movement parameters of these meters based on algorithmic methods. The article solves the problem of identification based on the maximum likelihood method, and theoretical estimates of identification errors are obtained. This makes it possible to estimate the state vector and parameters of the GSE movement in the presence of correlated disturbances of a deterministic and random nature.

The direction of further research can be the use of the obtained results for the construction of high-precision navigation and gravimetric systems.

Література

1. Безвесільна О.М. Прецизійний приладовий навігаційний комплекс та його чутливі елементи» / О.М. Безвесільна / КПІ ім. Ігоря Сікорського ДП НВЦ «Пріоритети». З грифом КПІ ім. Ігоря Сікорського. – Київ, 2020. – 451 с.

2. Безвесільна О.М. Оцінка стану гірогравіметра авіаційної гравіметричної системи з цифровою обробкою інформації : монографія / О.М. Безвесільна. – Київ : ДП НВЦ «Пріоритети», 2020. – 426 с.

3. Безвесільна О.М. Наукові дослідження в галузі автоматизації та приладобудування. Проектування та моделювання комп'ютеризованих інформаційно-вимірювальних систем : підручник з грифом ЖДТУ / О.М. Безвесільна, Ю.О. Подчашинський. – Київ : ДП «Редакція інформаційного бюлетеня «Офіційний вісник Президента України», 2021. – 896 с.

4. Безвесільна О.М. Методи планування та обробки результатів експериментів : підручник з грифом ЖДТУ / О.М. Безвесільна, Ю.О. Подчашинський. – Київ : ДП «Редакція інформаційного бюлетеня «Офіційний вісник Президента України», 2021. – 232 с.

5. Kordic V. Kalman Filter. Vukovar, Croatia: Intech, 2010. 390 p.

6. Безвесільна О. М. Вимірювання прискорень : підручник / О. М. Безвесільна. – К. : Либідь, 2001. – 264 с.

7. Основи та методи цифрової обробки сигналів: від теорії до практики: навч. посібник / уклад. : Ю.О. Ушенко, М.С. Гавриляк, М.В. Талах, В.В. Дворжак. – Чернівці : Чернівецький нац. ун-т ім. Ю. Федьковича, 2021. 308 с.

8. Пат. на винахід № 86005 України. МПК (2009) G01V 7/00, G01C 19/00. Гравіметр / О. М. Безвесільна, Ю.О. Подчашинський, І.В. Коробійчук ; заявник і патентовласник ЖДТУ. – № а 2005 04762 ;

заявл. 20.05.05 ; опубл. 25.03.09, Бюл. № 6.

References

1. Bezvesilna O.M. Precision instrument navigation complex and its sensitive elements" / O.M. Bezvesilna // KPI named after Ihor Sikorskyi State Enterprise "Priority" With the seal of KPI named after Igor Sikorsky. Kyiv. – 2020. – 451 p.

2. Bezvesilna O.M. Assessment of the state of the gyrogravimeter of the aviation gravimetric system with digital information processing: monograph / O.M. Bezvesilna - Kyiv: State Enterprise "Prioritety", 2020. – 426 p.

3. Bezvesilna O.M., Podchashinskyi Yu.O. Scientific research in the field of automation and instrumentation. Design and modeling of computerized information and measurement systems: Textbook with the handle of ZHTU / O.M. Bezvesilna, Yu.O. Podchashinskyi. – Kyiv: SE "Editorial Office of the Information Bulletin "Official Herald of the President of Ukraine", 2021. – 896 p.

4. Bezvesilna O.M., Podchashynskyi Methods of planning and processing the results of experiments: Textbook with the face of ZHTU / O.M. Bezvesilna, Yu.O. Podchashinskyi. – Kyiv: SE "Editorial of the information bulletin "Official Herald of the President of Ukraine", 2021. – 232 p.

5. Kordic V. Kalman Filter / V. Kordic. – Vukovar, Croatia: Intech, 2010. – 390 p.

6. Bezveselna O.M. Measurement of accelerations: textbook /O. M. Bezvesilna. – K.: Lybid, 2001. – 264 p.

7. Fundamentals and methods of digital signal processing: from theory to practice: teaching. manual / comp. : Yu.O. Ushenko, M.S. Havrylyak, M.V. Talakh, V.V. Dvořák – Chernivtsi: Chernivtsi national. University named after Yu. Fedkovicha, 2021. 308 p.

8. Pat. for the invention No. 86005 of Ukraine. IPC (2009) G01V 7/00, G01C 19/00. Gravimeter / O. M. Bezvesilna, Yu.O. Podchashinskyi, I.V. Korobiychuk; the applicant and the patent holder of ZHTU. – № a 2005 04762; statement 20.05.05; published 25.03.09, Bul. № 6.