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APPROXIMATION OF FREQUENCY DOUBLER MAGNETIZATION CURVES BASED ON A SECOND-ORDER POLYNOMIAL

The article is dedicated to the relevant problem of increasing the efficiency of ferromagnetic frequency doublers, which are important elements of modern high-frequency electronic systems. The relevance of the study is due to the growing demands in the modern world for performance, energy efficiency, and miniaturization of electronic devices. The aim of this work is to maximize the load current of a ferromagnetic frequency doubler. The optimization algorithm chosen is the genetic algorithm, which performs well in solving nonlinear problems and has the ability to find a global optimum in a complex multidimensional parameter space. A mathematical model of a three-rod ferromagnetic frequency doubler was used for the study. In this paper, a polynomial approach based on a second-order polynomial is proposed to approximate the magnetization curves, instead of the classical approach of approximation using a cubic spline. The cubic spline approximation has three sections: linear, nonlinear, and linear sections. The polynomial approach, in turn, contains only two sections: linear and nonlinear sections. In this approach, the coefficients of the polynomial are calculated relative to the end of the linear section, which is due to the need for the nonlinear section to continue where the linear section ends. One coefficient of the polynomial must be defined, for example, by a genetic algorithm, and the other two are calculated relative to the first. In this paper, the optimization was performed in three rounds. In the first round, the basic values of the frequency doubler were used, and the genetic algorithm searched for the polynomial coefficient. The second round additionally searched for input voltages. In the third round, in addition to the previous one, the search area was expanded to include the inverse inductances of the magnetic windings. The results obtained indicate a 15% increase in load current using the polynomial-based approximation compared to the cubic spline-based approximation.

Ключові слова: frequency doubler, mathematical model, genetic algorithm, parametric optimization.

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АПРОКСИМАЦІЯ КРИВИХ НАМАГНІЧУВАННЯ ПОДВОЮВАЧА ЧАСТОТИ НА ОСНОВІ ПОЛІНОМУ ДРУГОГО ПОРЯДКУ

Стаття присвячена актуальній проблемі підвищення ефективності феромагнітних подвоювачів частоти, які є важливими елементами сучасних високочастотних електронних систем. Актуальність дослідження зумовлена зростаючими вимогами до продуктивності, енергоефективності та мініатюризації радіоелектронних пристроїв. Метою роботи є максимізація струму навантаження феромагнітного подвоювача частоти. Оптимізаційним алгоритмом вибрано генетичний алгоритм, який добре показує себе у вирішенні нелінійних задач, а також має здатність знаходити глобальний оптимум у складному багатовимірному просторі параметрів. Для проведення досліджень використано математичну модель тристержневого феромагнітного подвоювача частота. У цій статті запропоновано використати поліном другого порядку для апроксимації кривих намагнічування, замість класичного підходу апроксимації із використанням кубічного сплайну. Апроксимація використовуючи кубічний сплайн має три ділянки: лінійну, нелінійну та лінійну ділянки. Поліноміальний підхід в свою чергу містить лише дві ділянки: лінійну та нелінійну ділянки. У такому підході коефіцієнти полінома розраховуються відносно кінця лінійної ділянки, що зумовлено потребою того щоб нелінійна ділянка продовжувалась там де закінчується лінійна. Один коефіцієнт полінома потрібно задати, наприклад генетичним алгоритмом, а інші два вираховуються відносно першого. У статті оптимізацію проведено у три раунди. В першому раунді використано базові значення подвоювача частоти, а генетичний алгоритм шукав коефіцієнт полінома. В другому раунді додатково відбувався пошук вхідних напруг. В третьому раунді додатково до попереднього область пошуку була розширена і включала обернені індуктивності магнітних віток. Отримані результати свідчать про зростання струму навантаження на 15% використовуючи апроксимацію на основі поліному у порівнянні із апроксимацією на основі кубічного сплайну.

Ключові слова: подвоювач частоти, математична модель, генетичний алгоритм, параметрична оптимізація.

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Problem statement and analysis of research and publications

In recent years, it has been difficult to ignore the rapid evolution of technology. A simple example is to look at our phones. 15 years ago, we couldn't imagine that we would end up in a situation where the modern phone would beat the laptop in processing power, where the phone's camera would overtake the sense of buying a dedicated camera for taking pictures, and there are many examples that we can highlight in retrospect.

This clearly shows the trend of devices becoming more complex while still trying to balance performance, size, power and cost [1]. As a result, companies are driven to improve overall efficiency and reduce power consumption. This is not limited to devices and varies across industries, from energy [2] to supply chains [3], as companies also strive to optimize their processes to achieve the best results [4].

In this article the attention will be focused on the frequency doubler, which is one of the important components in high-frequency electronics. It's main role is to take an input signal and produce an output at twice the frequency. It's widely used in RF [5] and microwave [6] systems, where the generation of signals is constrained by hardware limitations. By using frequency doubler the engineers can achieve higher frequencies without the need for more complex and as result more expensive oscillators [7]. Taking into account that performance of frequency doubler affects the quality of the whole system, it's good candidate for optimization, as this would help in achieving better results in efficiency, power consumption and other key factors.

There are several direction how frequency doubler can be optimized. In [8], the optimization is proposed based on the changing in the transformer core. In [9], optimization is done by simply trying three different predefined data sets on the three different circuits. In [10] is proposed algorithm-based optimization method which speed ups frequency doubler development from scratch. And in [11] is proposed genetic algorithm based approach for optimization.

An analysis of publications shows that researchers have been interested in conducting effective frequency doubler optimization in recent years, and emphasizes the importance of optimization to obtain more efficient devices in terms of power consumption, performance, and other metrics.

Formulation of the goals of the article

The purpose of the article is to optimize a three-rod ferromagnetic frequency doubler using a genetic algorithm. The optimization criterion is the maximization of the load current. To improve the results, it is proposed to approximate the magnetization curves using a second-order polynomial.

A mathematical model is used to perform the optimization. Although a mathematical model cannot completely replace experimental studies, the advantage of such a model is that it allows modeling different approaches and input data. As a result, a mathematical model can be used to conduct research that cannot be measured empirically in an experimental study.

Approximation of magnetization curves of a frequency doubler using a second-order polynomial

Ferromagnetic frequency multipliers multiply the frequency of the supply signal by a multiple of 2, 3, 4, ... Their design varies depending on this multiplier value. The reason why frequency multiplication occurs is the nonlinear dependence of the magnetic field strength of the multiplier cores on their induction.

In fact, a ferromagnetic frequency doubler is a magnetic amplifier with an additional winding. The second harmonic is induced in this additional winding, and the first harmonic is absent, just like in the control winding. Frequency doubler circuits are not new. In this article, we will use as a basis the mathematical model of a three-rod frequency doubler described in [11]. Therefore, this article will not repeat the formulas of the mathematical model, but the exception is when it is necessary to make changes to the original formula and the original formula will be given for reference. The schematic diagram is shown in Fig. 1.

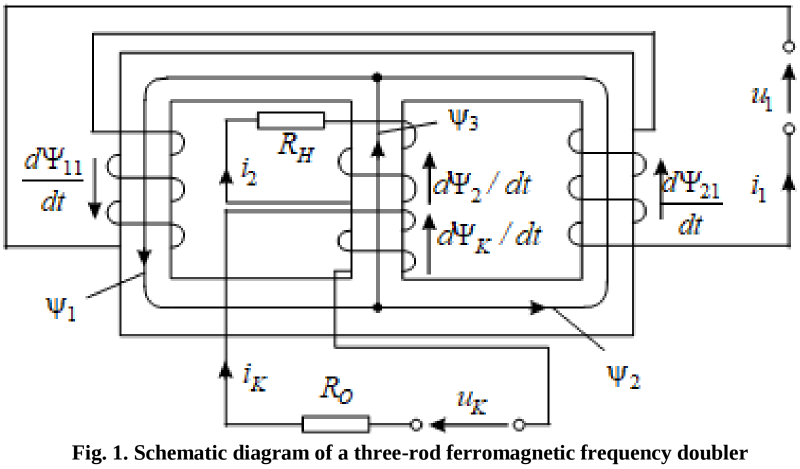

Fig. 1. Schematic diagram of a three-rod ferromagnetic frequency doubler

Formula (3) in [11] was used to calculate the magnetization curves, in this article, it will be labeled as (1) and presented below for clarity. If we analyze formula (1) in more detail, we can see that there are 3 general sections. The first and third sections are linear, the second is nonlinear.

$$\varphi(\psi) = \begin{cases} a_1\psi, & |\psi| > \psi_{01}, \\ S_3(\psi), & \psi_{01} \le |\psi| \le \psi_{02}, \\ a_2\psi - a_0, & |\psi| > \psi_{02} \end{cases} \qquad (1)$$

where $S_3(\psi)$ – cubic spline; $a_j (j = 0, 1, 2)$ – approximation coefficients.

The magnetization curves of the magnetic cores are taken to be the same and approximated by expression (1) with the choice of the calculation formula, where a_1 = 1.0 H^{-1}; a_2 = 50 H^{-1}; a_0 = 43 A; ψ_{01} = 0.5 Wb; ψ_{02} = 1.1 Wb; $\varphi(\psi_{01})$ = 0.5 A; $\varphi(\psi_{02})$ = 12 A.

Let us introduce (2) to calculate the second-order polynomial:

$$p_2(\psi) = b_0 + b_1\psi + b_2\psi^2 \qquad (2)$$

where b_0, b_1, b_2 – polynomial coefficients.

It is proposed to use (2) to calculate the second, nonlinear section, and the third, linear section is eliminated. However, an important nuance is that in order to calculate the polynomial, it is necessary to fix the value at the end of the linear section, which in this case is ψ_{01} = 0.5 Wb. To do this, one can use the coefficient search approach, when the coefficient b_2 is set to a certain fixed value, and the other two coefficients b_0, b_1 are expressed relative to ψ_{01}. First, the coefficient b_1 needs to be expressed:

$$b_1 = a_1 - 2b_2\psi_{01} \quad (3)$$

When the coefficient b_1 is known, the only thing left to do is to express b_0:

$$b_0 = \varphi(\psi_{01}) - b_1\psi_{01} - b_2{\psi_{01}}^2 \quad (4)$$

Substituting (2) into formula (1), we obtain:

$$\varphi(\psi) = \begin{cases} a_1\psi, & |\psi| < \psi_{01}, \\ p_2(\psi), & |\psi| \geq \psi_{01} \end{cases} \quad (5)$$

If the coefficient b_2 = 35, then after the calculation the values b_1 = -34.0 and b_0 = 8.75, and all other values for approximating the magnetization curves will be the same as those mentioned at the beginning of this section. Fig. 3 shows the approximation on the interval $\psi \in [0, 2]$ Wb using (5) for approximation.

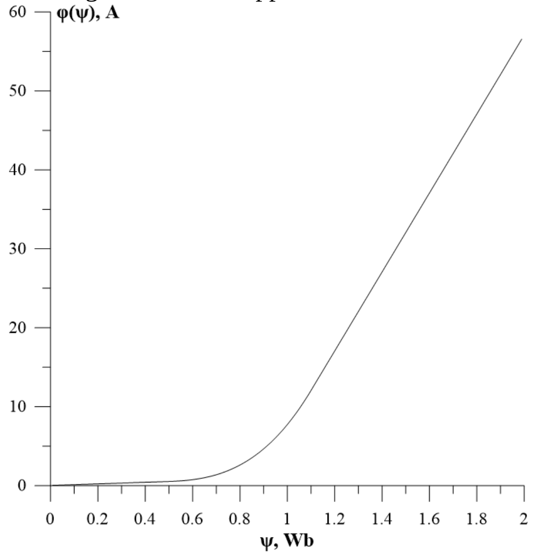

Fig. 2. Approximation of the magnetization curve using a cubic spline

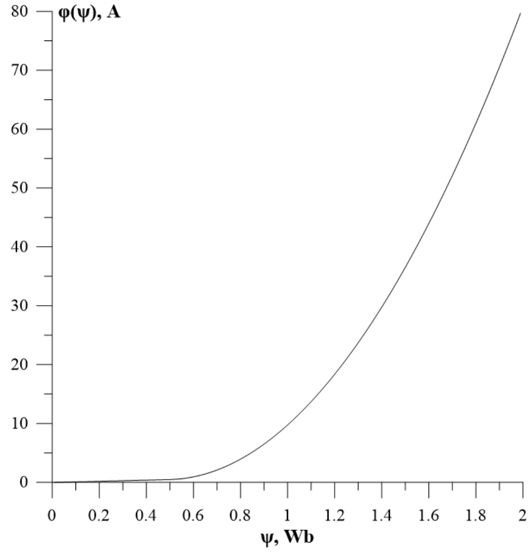

Fig. 3. Approximation of the magnetization curve using a second-order polynomial

Conducting optimization

The Python library PyGAD [12] is used to perform the optimization, which provides an extensive implementation of the genetic algorithm with full customization. This library is open-source, and it is still supported with periodic updates that fix bugs and/or provide new functionality. In this article, the optimization goal is to maximize the value of the load current i_2.

Let's use the same parameters as in [11] for the initial calculations before optimization $r_{11} = r_{21}$ = 1.5 Ohm; $R_0 = r_C = r_2$= 3 Ohm; R_H = 17 Ohm; $\alpha_{11} = \alpha_{21}$ = 120 H^{-1}; α_2 = 170 H^{-1}; α_C = 100 H^{-1}; u_C = 100 V. The supply voltage is set by the equation $u_1 = U_m \sin(\omega t)$, where U_m = 537.4 V, ω = 314.1593 rad/s. To approximate the magnetization curves, we use (1) and the same parameters as at the beginning of the previous section. As is already known from [11], using these parameters, the maximum value of i_2 = 1.178 A is obtained. Fig. 4 shows the steady-state values of the load current i_2.

Before proceeding with the optimization, it is necessary to set up the genetic algorithm (GA), in this article the following configuration is used: number of parents for crossing = 2, number of individuals per generation = 20, number of generations = 60, and parental selection through rank. Since the goal of optimization is to maximize the value of i_2, this value will be selected in the interval for one period of the supply voltage in steady state, namely 0.02 s. Accordingly, the fitness function will have the following value:

$$fitness = \max(i_2) \quad (6)$$

Each individual from the population provides values generated in a given search area, so it can be considered that one individual is one potential solution. The model accepts configuration as an input parameter, so to integrate the model into a genetic algorithm, it is necessary to modify the configuration before performing calculations in the model, and the model itself does not require any changes.

To approximate the magnetization curves, let us use (5), and to find the coefficient b_2 let us use a genetic algorithm, reminding that the coefficients b_1 and b_0 are expressed according to (3) and (4), respectively. The search area is set to $b_2 \in [0, 50]$. This means that each individual will provide a solution for only one parameter within the given range. After conducting optimization, the individual with the best fitness value is obtained with the following value b_2 = 49.32, and the values b_1 = -48.32 and b_0 = 12.33 were calculated accordingly. It can be noted that the value of b_2 has reached the boundary of the search area, but in this study we leave the search area unchanged, noting that there is room for further research. Fig. 5 shows the results of the first optimization. The maximum value of the load current was i_2 = 1.376 A.

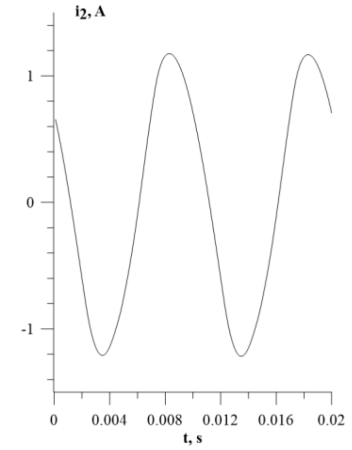

Fig. 4. Steady-state load current values before optimization

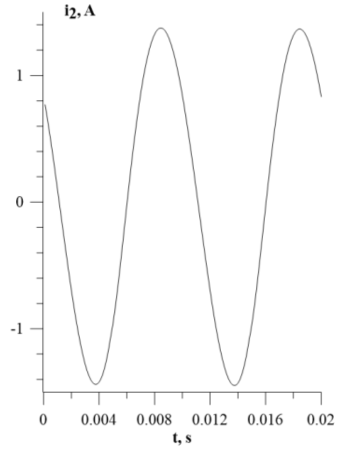

Fig. 5. Steady-state load current values after the first optimization

The second optimization will be performed with additional search intervals for the parameters of the supply voltage u_1, which varies relative to U_m and the control voltage u_C. Let's set the following search intervals U_m ∈ [220, 800] V and u_C ∈ [50, 220] V, and leave the search interval b_2 ∈ [0, 50] unchanged. After conducting optimization, the following values were obtained: u_C = 56.18 V; U_m = 795.03 V and b_2 = 49.76, and the calculated values of b_1 = -48.76 and b_0 = 12.44. The maximum value of i_2 = 2.685 A was reached. Note that the search interval U_m could be further expanded, however, current trends are just started moving towards the introduction of 800 V architecture for electric vehicle batteries [13]. It is expected that in the coming years, 800 V architecture will gradually replace 400 V architecture [14], so it does not make much sense to increase the search interval above 800 V at this time, as the application areas for the device will be greatly reduced. Figure 6 shows the results of the optimization.

In addition to changing the input voltages, the inverse of the windings' dissipation inductances can also be added to the search space, because these parameters change as a result of a design change in the parameters of the magnetic cores. For the third optimization, let's define the search space: U_m ∈ [220, 800] V; u_C ∈ [50, 220] V and b_2 ∈ [0, 50] without changes. Let's also introduce additional search intervals: $\alpha_{11} = \alpha_{21}$ ∈ [70, 170] H^{-1}; α_2 ∈ [120, 220] H^{-1} and α_K ∈ [50, 150] H^{-1}. After conducting optimization the obtained values are: u_C = 95.71 V; U_m = 791.04 V; $\alpha_{11} = \alpha_{21}$ = 119.43 H^{-1}; α_2 = 215.09 H^{-1}; α_K = 51.03 H^{-1} and b_2 = 49.92, respectively, the values are calculated b_1 = -48.92 and b_0 = 12.48. As a result, the maximum value of i_2 = 3.429 A was obtained. The optimization results are shown in Fig. 7.

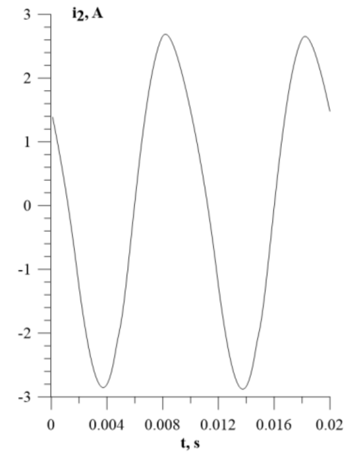

Fig. 6. Steady-state load current values after the second optimization

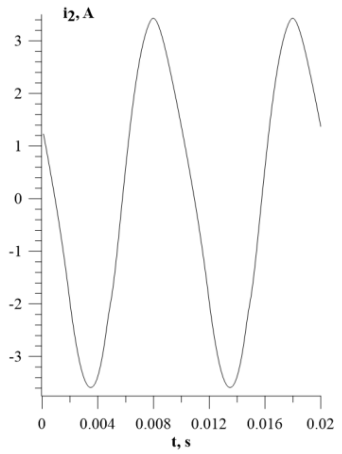

Fig. 7. Steady-state load current values after the third optimization

It is worth comparing the maximum values of the load current i_2 obtained using the cubic spline and using a second-order polynomial to approximate the magnetization curves. It should be noted that the optimization data using the cubic spline are taken from the results of the third and fourth optimization described in [11], namely i_2 = 2.087 A and i_2 = 2.936 A, respectively. Simply taking the optimized parameters from this article and using the cubic spline approximation of the magnetization curves would not be entirely correct, since these two approximation methods are different. Also, because the genetic algorithm finds the best solutions relative to the environment in which the optimization is performed, therefore it does not mean that the best values found for one method will be also the best for the other.

Table 1

Comparison of the maximum value of the load current obtained in different optimization scenarios and methods of approximating magnetization curves

Optimization scenario	Cubic spline	Second-order polynomial
Parameters before optimization	i_2 = 1.178 A	i_2 = 1.376 A
Parameters after voltage optimization	i_2 = 2.087 A	i_2 = 2.685 A
Parameters after optimization of voltage and inverse of the windings' dissipation inductances	i_2 = 2.936 A	i_2 = 3.429 A

After analyzing the data, it can be seen that the polynomial approach to approximating the magnetization curves resulted in an increase of just over 15%.

Conclusions from this research and prospects for further research in this direction

In this study, a polynomial approach to approximating magnetization curves is proposed, which uses a second-order polynomial instead of a cubic spline. In contrast to the cubic spline-based approach, the polynomial approach has only two sections, a linear section and a nonlinear section. As a result, the magnetic characteristics of the polynomial approach are smoother and the output signal is very close to or replicates a sinusoidal shape, which is more difficult to achieve with the cubic spline approach without sacrificing the power of the output signal.

Three rounds of optimization were performed. The first round used the basic values and searching the polynomial coefficients using a genetic algorithm, the second round additionally introduced search of the voltage values, and in the third round additionally searched the values of the inverse inductances of the windings. Compared to the optimization using a cubic spline, the polynomial approach shows better results by slightly more than 15%. The obtained results demonstrate positive dynamics in the implementation of the polynomial approach for the approximation of magnetization curves.

Further research can be directed in the following areas:

- Extend the optimization to other nonlinear circuits, such as balanced or push-pull frequency doublers, magnetic amplifiers, etc...
- Use of multi-objective optimization, which allows finding a compromise solution rather than a single optimal point.
- Hybrid optimization approaches that combine the best of different optimization methods, which can improve convergence rates and/or provide better solutions. [15]
- Experimental validation on a physical prototype to assess the real-world applicability and improve the model.
- Integration of machine learning models, such as predictive models trained on simulation data, can speed up computations and, consequently, the speed of optimization.

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