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SOLUTION OF NONLINEAR OPTIMIZATION PROBLEMS OF HEAT TRANSFER THEORY

The article proposes mathematical models and computational methods for the solution of nonlinear problems of finding local extrema of the objective function. These mathematical models and computational methods create a computational structure for improving the accuracy of applied optimization problems. Computational mathematical models of thermal action on a multilayer material and laser action on an embryo have been developed. It should be noted that the computational mathematical model describing the laser effect on the embryo is a nonlocal boundary value problem with a system of evolutionary, nonstationary heat conduction equations, heat flow boundary conditions, and boundary conditions at the beginning and end of the laser effect on the embryo. Applying the traditional theory of existence and uniqueness of the solution of the boundary value problem, due to the multilayer structure of the embryo and different thermal modes of laser action without averaging the values of the thermophysical characteristics of the object of study, it is impossible to substantiate the correctness of the boundary value problem describing the laser action on the embryo. To substantiate the correctness of this boundary value problem, the article proposes to apply methods from the theory of pseudo-differential operators in the space of slowly increasing distributions that depend smoothly on time variables. This made it possible not only to improve the accuracy of solving boundary value problems, but also to increase the accuracy of solving the general problem of improving the quality of laser action on a multilayer material.

Reducing the time required to solve boundary value problems was achieved through the use of functionally oriented blocks that almost instantly implement many computational mathematical models (boundary value problems) on computers. This will increase the efficiency of existing technical means and develop new technical means for automating the design of complex systems containing local sources of thermal action.

Keywords: mathematical models, computational methods, function-oriented blocks, automation.

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РОЗВ'ЯЗАННЯ НЕЛІНІЙНИХ ОПТИМІЗАЦІЙНИХ ЗАДАЧ ТЕОРІЇ ТЕПЛОПЕРЕНЕСЕННЯ

В статті запропоновані математичні моделі і обчислювальні методи для розв'язання нелінійних задач пошуку локальних екстремумів функції мети. Ці математичні моделі і обчислювальні методи створюють обчислювальну структуру для підвищення точності реалізації прикладних оптимізаційних задач. Розроблені розрахункові математичні моделі термічної дії на багатошаровий матеріал і лазерної дії на ембріон. Зазначимо, що розрахункова математична модель, яка описує лазерну дію на ембріон – це нелокальна крайова задача з системою еволюційних, нестаціонарних рівнянь теплопровідності, граничними умовами теплового потоку і граничними умовами на початку і наприкінці лазерної дії на ембріон. Застосувавши традиційну теорію існування і єдиності розв'язку крайової задачі, через багатошарову будову ембріона і різні теплові режими лазерної дії без усереднення значень теплофізичних характеристик об'єкта дослідження неможливо обґрунтувати коректність крайової задачі, що описує лазерну дію на ембріон. Для обґрунтування коректності цієї крайової задачі в статті запропоновано застосувати методи з теорії псевдодиференціальних операторів в просторі повільно зростаючих розподілів, які гладко залежать від часових змінних. Це дозволило не лише підвищити точність розв'язання крайових задач, а також і збільшити точність розв'язання загальної задачі підвищення якості лазерної дії на багатошаровий матеріал.

Зменшення часових витрат для розв'язання крайових задач досягнуто завдяки застосуванню функціонально-орієнтованих блоків, які практично миттєво реалізують безліч розрахункових математичних моделей (крайових задач) на комп'ютерах. Це дозволить підвищити ефективність вже існуючих технічних засобів і розробити нові технічні засоби для автоматизації проектування складних систем, які містять локальні джерела термічної дії.

Ключові слова: математичні моделі, обчислювальні методи, функціонально-орієнтовані блоки, автоматизація.

Formulation of the problem

When solving the general problem of calculating and optimizing the parameters of complex systems, many scientists go by solving specific problems. This means that they select specific systems with certain parameters to be optimized in advance, and average the values of the remaining system parameters, which are usually neglected when implementing computational and applied optimization mathematical models. Specific mathematical models are built for private tasks, and after their implementation, the values of the above parameters are obtained. In the case of changes in optimized systems, studied objects under the influence of load sources of physical fields or optimized technical parameters, other computational methods must be used to obtain an analytical (approximate-analytical) solution to boundary value problems. It should be noted that the choice of methods for calculating and optimizing the control parameters of systems mostly depends on such features of the modeled systems as: the shape of the object of study and structural features (object structure, presence of areas of prohibition on the influence of physical field sources), technical characteristics of the sources of influence and permissible modes of influence on the object under study. Thus, when changing the system whose parameters are being optimized, it is necessary to change the calculation and optimization methods.

The article proposes an approach to the development of specialized modeling computing devices that can improve the efficiency (in terms of accuracy, time and memory) of solving problems of finding optimal parameters of the technological process of laser exposure to multilayer microbiological materials. And this, in turn, will make it possible to obtain hardware to automate the design of complex systems containing local sources of thermal effects.

Analysis of the latest research

The results of the publication [1] are devoted to the solution of multimodal optimization problems of high dimensionality. Having carried out a comprehensive detailed analysis of the methods used to solve optimization problems, the authors of [1] proposed criteria for determining effective solution methods. Articles [2–4] proposed methods for solving not particular problems, but a whole class of mathematical programming problems with uncertain input data. To reduce the consumption of energy resources and improve the state of the energy system at Ukrainian industrial enterprises, the mathematical model of thermal and gas-dynamic processes in a steam turbine was improved in [5] to take into account the specific features of the modeled systems. The authors of [6] proved the correctness of a nonlocal boundary value problem for a parabolic equation of finite and infinite orders over the space of generalized functions. The solutions of applied geometric design problems obtained in articles [7–9] allowed their authors to develop effective strategies for the functioning of individual biotechnological and technical systems. Direct and inverse approximation theorems in the spaces of periodic and almost periodic functions were obtained [10, 11]. The article [12] calculated the optimal modes of metal melting in metallurgy.

The purpose of the work is development of mathematical models and methods for solving nonlinear optimization problems of heat transfer theory.

Presenting main material

We present a computational structure for calculating and optimizing the technical parameters of the impact on the material:

– method for solving the boundary value problem for regions of complex spatial shape based on V.L. Rvachova functions (RFM method);
– parameterization of solving the boundary value problem;
– approximate gradient method for finding local extrema by thermal influence parameters;
– method of directed search for local extremes.

The main optimization task is to propose the principles of creating software and hardware to reduce the consumption of the material under study during thermal exposure to it by taking into account the restrictions on the resulting temperature field and the technical characteristics of the emitters [13, 14]. The paper presents the boundary value problem of the process of thermal impact on a multilayer spherical material and its approbation for the process of laser impact on an embryo. The system of differential equations of thermal conductivity:

$$\begin{cases} \rho_1 c_1 \dfrac{\partial T_1}{\partial t} - \lambda_1 \left(\dfrac{\partial^2 T_1}{\partial r^2} + \dfrac{2}{r_1} \dfrac{\partial T_1}{\partial r} \right) + q_1 = 0, \quad r \in [0; r_1],\ t \in [0; t_1]; \\ \rho_2 c_2 \dfrac{\partial T_2}{\partial t} - \lambda_2 \left(\dfrac{\partial^2 T_2}{\partial r^2} + \dfrac{2}{r_2} \dfrac{\partial T_2}{\partial r} \right) + q_2 = 0 \quad r \in [r_1; r_2],\ t \in [t_1; t_2]; \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \rho_N c_N \dfrac{\partial T_N}{\partial t} - \lambda_N \left(\dfrac{\partial^2 T_N}{\partial r^2} + \dfrac{2}{r_N} \dfrac{\partial T_N}{\partial r} \right) + q_N = 0 \quad r \in [r_{N-1}; r_N],\ t \in [t_{N-1}; t_N], \end{cases} \tag{1}$$

where ρ_e – density coefficient e-th layer of the multilayer (N-layered) material, where $e = 1,..,N$;
c_e – heat capacity coefficient e-th of the multilayer material;
λ_e – heat transfer coefficient e-th layer of multilayer material;
$T_e = T_e(r,t)$ – temperature field;
r_e – distance from the center of the thermal source to the point in the e-th temperature field value is calculated in the layer of the multilayer material.
q_e – specific power density of heat loads in multilayer material.

To take into account the beginning and the end of thermal effects, we will use Dirichlet boundary conditions:

$$\begin{cases} T(r,t)\big|_{t=t_0}^{r=r_0} = T_0; \\ T(r,t)\big|_{t=t_n}^{r=r_n} = T_n, \end{cases} \tag{2}$$

where T_0 – material temperature Ω at first heat exposure;

T_n – material temperature Ω at the end of thermal exposure.

To account for the multilayered (*N*-layered) of the material structure, we introduce the interface equality and the continuity equality of the temperature fields. Equalities of the interface:

$$\begin{cases} T_1(r_1,t_1) = T_2(r_2,t_2), \quad -\lambda_1 \dfrac{\partial T_1}{\partial r} = -\lambda_2 \dfrac{\partial T_2}{\partial r}, \quad r \in [r_1; r_2]; \\ T_2(r_2,t_2) = T_3(r_3,t_3), \quad -\lambda_2 \dfrac{\partial T_2}{\partial r} = -\lambda_3 \dfrac{\partial T_3}{\partial r}, \quad \in [r_2; r_3]; \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ T_{N-1}(r_{N-1},t_{N-1}) = T_N(r_N,t_N), \quad -\lambda_{N-1} \dfrac{\partial T_{N-1}}{\partial r} = -\lambda_N \dfrac{\partial T_N}{\partial r}, \quad r \in [r_{N-1}; r_N]. \end{cases} \tag{3}$$

Equalities of continuity of temperature fields along the time coordinate on layers of multilayer (*N*-layer) material:

$$\begin{cases} T(r_1; t_1 - 0) = T(r_1; t_1 + 0); \\ T(r_2; t_2 - 0) = T(r_2; t_2 + 0); \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ T(r_N; t_N - 0) = T(r_N; t_N + 0). \end{cases} \tag{4}$$

Heat flow boundary conditions at the interface between the outer layer of the material and the environment:

$$-\lambda_1 \frac{\partial T_1}{\partial r}(0,t) = qS, \quad 0 \le t \le h, \tag{5}$$

where q – specific heat flux on the outer layer of the material Ω;

S – the area of the source of exposure in the form of a spot.

The system of heat conduction equations from the boundary value problem of systems of differential heat conduction equations for laser treatment of the embryo:

$$\begin{cases} \rho_1 c_1 \dfrac{\partial T_1}{\partial t} - \lambda_1 \left(\dfrac{\partial^2 T_1}{\partial r^2} + \dfrac{2}{r_1} \dfrac{\partial T_1}{\partial r} \right) + q_1 = 0; \\ \rho_2 c_2 \dfrac{\partial T_2}{\partial t} - \lambda_2 \left(\dfrac{\partial^2 T_2}{\partial r^2} + \dfrac{2}{r_3} \dfrac{\partial T_2}{\partial r} \right) + q_2 = 0; \\ \rho_3 c_3 \dfrac{\partial T_3}{\partial t} - \lambda_3 \left(\dfrac{\partial^2 T_3}{\partial r^2} + \dfrac{2}{r_3} \dfrac{\partial T_3}{\partial r} \right) + q_3 = 0; \\ \rho_4 c_4 \dfrac{\partial T_4}{\partial t} - \lambda_4 \left(\dfrac{\partial^2 T_4}{\partial r^2} + \dfrac{2}{r_4} \dfrac{\partial T_4}{\partial r} \right) + q_4 = 0. \end{cases} \tag{6}$$

The Dirichlet boundary conditions (2) define the beginning and the end of laser action on the embryo. To take into account the three-layer structure of the embryo, the partition equilibria and the continuity equilibria of the temperature fields are applied. The partition equilibria have the following form:

$$\begin{cases} T_1(r_1,t_1) = T_2(r_2,t_2), \quad -\lambda_1 \dfrac{\partial T_1}{\partial r} = -\lambda_2 \dfrac{\partial T_2}{\partial r}, \quad r \in [r_1; r_2]; \\ T_2(r_2,t_2) = T_3(r_3,t_3), \quad -\lambda_2 \dfrac{\partial T_2}{\partial r} = -\lambda_3 \dfrac{\partial T_3}{\partial r}, \quad r \in [r_2; r_3]; \\ T_3(r_3,t_3) = T_4(r_4,t_4), \quad -\lambda_3 \dfrac{\partial T_3}{\partial r} = -\lambda_4 \dfrac{\partial T_4}{\partial r}, \quad r \in [r_3; r_4]; \\ T_4(r_4,t_4) = T_5(r_5,t_5), \quad -\lambda_4 \dfrac{\partial T_4}{\partial r} = -\lambda_5 \dfrac{\partial T_5}{\partial r}, \quad r \in [r_4; r_5]; \\ T_5(r_5,t_5) = T_6(r_6,t_6), \quad -\lambda_5 \dfrac{\partial T_5}{\partial r} = -\lambda_6 \dfrac{\partial T_6}{\partial r}, \quad r \in [r_5; r_6]. \end{cases} \tag{7}$$

The continuity equations for the temperature fields will be in the form (4), taking into account the three-layer structure of the embryo. Boundary conditions (5) define the heat flux on the outer layer of the embryo. By applying methods from the theory of pseudo-differential operators in the space of slowly increasing distributions that smoothly depend on time variables, the correctness of the above boundary value problems is substantiated. To solve the boundary value problem (6)–(7), the authors propose to use the Fourier method of separated variables [13, 15]. Let us consider one of the possible approaches to the development of specialized modeling computational devices that allow to increase the efficiency (in terms of accuracy, time and memory consumption) of solving problems of searching for optimal parameters of the technological process of laser fission of multilayer microbiological materials. And this, in its turn, will give an opportunity to obtain hardware to automate the process of research of such mathematical models.

Let's characterize the main blocks (1 – block of setting parameters of the mesh model of the area of the material to be cut; 2 – mesh model of the area of the material to be cut; 3 – block of setting initial, boundary conditions and interface conditions between layers; 4 – block of setting permissible values of the temperature field; 5 – comparison block; 6 – block of calculating laser parameters; 7 – block of searching rational laser parameters; 8 – block of setting laser parameters; 9 – block of input-output information) necessary for the implementation of the main optimization problem. For the preparation of initial data on the stage of development, structure and geometric characteristics of microbiological material a microscope connected to a television camera, the signals of which are fed to the computer input, is necessary. This makes it possible to obtain the necessary initial information to perform modeling procedures and search computational process. Besides, on the basis of these data with the help of the program «Tracing» a set of admissible trajectories of movement of the laser beam, i.e., a spot at thermal influence on microbiological material is determined. It should be noted that at this preliminary stage a set of rational traces is formed taking into account only the structure of microbiological material and its geometric characteristics. In the future, this set of traces, as a rule, is narrowed down due to additional restrictions on the temperature field of the microbiological material [13, 15, 16].

It is also necessary to include in the hardware blocks modeling the spectrum of the corresponding boundary value problems. Moreover, it is reasonable to use analog or analog-to-digital grid processors with variable structure and parameters to implement boundary value problems and to significantly reduce time costs compared to computers. The use of analog grid processors will allow to realize almost instantly any complex boundary value problem. This makes it possible to minimize the time of solving the corresponding boundary value problem at each iteration of the process of searching for the parameters of thermal influence. In this case, the time of realization of the stage of the boundary value problem solution will be determined mainly only by the time of input of initial information and preparation of the device for work. To implement the procedure of setting the initial parameters of thermal exposure it is advisable to use appropriate methods and devices to determine the size of the area of multilayer microbiological material, the intensity of the laser source, i.e., the spot, the energy of thermal exposure, the location of the laser spot, the speed of movement of the laser spot on the microbiological material, the density of thermal exposure, the diameter of the spot. In order to realize the procedure of parameters evaluation for their admissibility it is advisable to provide a block of comparison of values of the temperature field of microbiological material in the investigated point at the given moment of time and a given admissible value.

In addition, it is advisable to provide a connection between the comparison block and the block for calculating the technical parameters of the radiators. Through this connection, the signal of the mismatch value between the temperature field values obtained on the grid model and the specified permissible value will arrive at the input of the parameter calculation block. Depending on this signal is decided on the admissibility of the parameters obtained at this iteration or their inadmissibility. To implement the procedure of searching for optimal values of the sought parameters of thermal action, on the set of admissible, it is advisable to use blocks that implement search methods of optimization, for example, the step method. In order to realize the final operation of assessing the viability of parts of microbiological material, it is advisable to check the fulfillment of the differential (integral) criterion of embryo viability given in work [15].

Preparation of the device for operation begins with input of initial information into block 9. Then from block 9 to the input of block 4 comes the permissible value of the controlled physical parameter, for example, the permissible value of the temperature field in the points of a sufficiently small neighborhood of the laser spot boundary. The initial values of the corresponding parameters for their setting on the mesh model 2 are supplied to block 1 of setting the parameters of the mesh model. From block 9 to the input of block 6 of calculation of laser parameters the information on initial values of parameters of laser beam action on microbiological material, namely: intensity of laser beam source, i.e., spot, energy and duration of thermal effect, trajectory and speed of laser source movement, density of thermal effect, diameter of laser source, i.e., spot, is received. The input of block 3 receives information about initial, boundary conditions and conditions of conjugation between layers in a multilayer microbiological material. This completes the preparation of the device for work.

Initial laser parameters from block 6 through block 7 are fed to block 8 and then to grid model 2 of the thermal impact area, which makes it possible to simulate the effect of the laser beam on microbiological material. The values of the temperature field of the controlled points of the grid model 2 come to the input of block 5, where the comparison with the permissible temperature value set earlier from block 4 is carried out. Block 5 compares the set permissible value of the temperature field with that obtained on the grid model 2. If the temperature field on the grid model 2 is higher than the predetermined one, then from block 5 a signal is sent to block 6 of the laser parameters calculation, which is used to correct the laser parameters, and block 9 of the registration records the corresponding laser parameters. Thus, for several iterations according to the set restrictions on the value of the of the temperature field in the points of sufficiently small vicinity of the laser spot boundary, the optimal values of the laser beam parameters are determined, namely: laser beam source intensity, energy and duration of thermal impact, trajectory and velocity of the laser source, density of thermal impact, diameter of the laser source.

Conclusions

The research article is devoted to the solution of nonlinear optimization problems of heat transfer theory in complex systems. The generality of the method and the basic algorithm for solving the main problem of improving the quality of exposure by reducing the consumption of the material under study is proposed. This is achieved by taking into account the limitations both on the resulting temperature field and on the technical parameters of the impact sources. The permissible values of technical parameters of the emitters are taken from the technical characteristics of the devices. The computational structure for calculation and optimization of parameters of technical systems containing sources of thermal loading is given. The authors propose to use the gradient method to search for local extrema of the temperature field values. Using the method of directed enumeration of local extrema of the target function, the pre-selected optimal technical parameters of laser emitters are obtained. Using the research results of this article to solve applied optimization problems allowed us to obtain not only hardware and software tools for automating the design of complex systems containing sources of thermal loading, but also to reduce time costs in the implementation of the main task of improving the quality of thermal effects, as well as minimizing the damage to the material under study.

References

1. Kosolap, A. (2023). Rozviazuvannia multymodalnykh optymizatsiinykh zadach velykoi rozmirnosti [Solving multimodal optimization problems of high dimensionality]. *Fizyko-matematychne modeliuvannia ta informatsiini tekhnologii*, (36), 126–130. https://doi.org/10.15407/fmmit2023.36
2. Raskin, L., Sira, O., Hatunov, A., Riabokon, R., & Sinitsyn, R. (2024). Method for solving distributional problems of mathematical programming under conditions of fuzzy initial data. *Eastern-European Journal of Enterprise Technologies*, 4(4(130)), 63–68. https://doi.org/10.15587/1729-4061.2024.310569
3. Kyrychenko, I., Tereshchenko, G., & Smelyakov, K. (2024). Optimized indexing method in a hybrid image storage model for efficient storage and access in big data environments. In *2024 IEEE 17th International Conference on Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET)* (pp. 1–4). Lviv. https://doi.org/10.1109/TCSET64720.2024.10755763
4. Pavlichkov, S., & Bajcinca, N. (2024). Small gain conditions for stability of infinite networks of time-delay systems and applications. In *2024 European Control Conference (ECC)* (pp. 2386–2392). Stockholm, Sweden. https://doi.org/10.23919/ECC64448.2024.10590849
5. Usatyi, A. P., & Klob, O. P. (2024). Podalshe udoskonalennia matematychnoi modeli termo ta hazodynamichnykh protsesiv v turbini bez robochykh lopatok chastyny stupeniv [Further improvement of the mathematical model of thermal and gas-dynamic processes in a turbine without working blades of part of the stages]. *Visnyk Natsionalnoho tekhnichnoho universytetu «KhPI». Seriia: Enerhetychni ta teplotekhnichni protsesy y ustatkuvannia*, (1), 11–20. https://doi.org/10.20998/2078-774X.2024.01.02
6. Horodets'kyi, V., Petryshyn, R., & Martynyuk, O. (2023). Evolutionary pseudodifferential equations with smooth symbols in the S-type spaces. *Ukrains'kyi Matematychnyi Zhurnal*, 75(6), 753–776. https://doi.org/10.37863/umzh.v75i6.7443
7. Fardigola, L., & Khalina, K. (2022). Controllability problems for the heat equation in a half-plane controlled by the Dirichlet boundary condition with a point-wise control. *Journal of Mathematical Physics, Analysis, Geometry*, 18(1), 75–104. https://doi.org/10.15407/mag18.01.075
8. Zhuchenko, L. K. (2022). Postanovka zadachi optymalnogo keruvannia protsesom vypaliuvannia vugletsevykh vyrobiv [Formulation of the problem of optimal control of the carbon products firing process]. *Vcheni zapysky Tavriiskogo Natsionalnogo Universytetu imeni V.I. Vernadskogo. Seriia: «Tekhnichni nauky»*, 33(72)(5), 81–85. https://doi.org/10.32782/2663-5941/2022.5/11
9. Ladiieva, L. R., Korniienko, B. Ia., & Pylypon, O. V. (2024). Optymalne keruvannia protsesom paro-kysnevoi konversii metanu [Optimal control of the process of steam-oxygen conversion of methane]. *Prykladni pytannia matematychnoho modeliuvannia*, 7(1), 164–174. https://doi.org/10.32782/mathematical-modelling/2024-7-1-15
10. Serdyuk, A., & Shidlich, A. (2024). Actual problems of the theory of approximations in metrics of discrete spaces on the sets of summable periodic and almost periodic functions. *Ukrains'kyi Matematychnyi Zhurnal*, 76(11), 1653–1690. https://doi.org/10.3842/umzh.v76i11.8525
11. Sklyar, G., Barkhayev, P., Ignatovich, S., & Rusakov, V. (2022). Implementation of the algorithm for constructing homogeneous approximations of nonlinear control systems. *Mathematics of Control, Signals, and Systems*, 34, 883–907. https://doi.org/10.1007/s00498-022-00330-5
12. Zakharov, A. V., Rybalko, I. M., & Saichuk, O. V. (2023). Metalurhiini protsesy plavlennia i perenesennia elektrodnoho ta prysadnoho materialiv u shlakovii vanni pry elektroshlakovomu naplavlenni [Metallurgical processes of melting and transfer of electrode and filler materials in the slag bath during electroslag surfacing]. *Visnyk LTEU. Seriia: «Tekhnichni nauky»*, (33), 12–18. https://doi.org/10.36477/2522-1221-2023-33-02
13. Levkin, A., Abuselidze, G., Berezhna, N., Levkin, D., Volkova, T., & Kotko, Y. (2022). The quality function in determining the effectiveness of example bioeconomics tasks. *Rural Sustainability Research*, 48(343), 91–102. https://doi.org/10.2478/plua-2022-0019
14. Levkin D. Mathematical modeling of the process of containerized cargo handling at terminals. / Levkin D., Berezhna N., Kozenok A., Babych I. // Central Ukrainian Scientific Bulletin. Technical sciences. – 2023. – Issue. 8 (39). Part. 1. – S. 196–201. https://doi.org/10.32515/2664-262X.2023.8(39).1.196-201

15. Kavun S. Methods of Mathematical Programming for Designing a Safe Environment for Bioobject. / Kavun S., Levkin D., Levkin A., Kotko Y., Levkina R. // CEUR Workshop Proceedings. (26.10.2023). – Kyiv, 2023. – No. 3550. – Pp. 255–260.

16. Levkin D. Improvement of the methodology for optimizing the parameters of complex systems under the influence of thermal load sources. / Levkin D. // Herald of Khmelnytskyi National University. Technical Sciences. – Khmelnytskyi, 2024. – Vol. 331. No 1. – S. 69–72. https://doi.org/10.31891/2307-5732-2024-331-11